

AshCalc

A tephra volume calculations tool

User manual

NERC Research Experience Placement Scheme

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Requirements

AshCalc is written in Python 3 and therefore should be compatible with Windows, Linux and Mac operating systems.

Requirements for running AshCalc are:

- Python 3
- NumPy (version 1.7.1 or greater)
- SciPy (version 0.12.0 or greater)
- Matplotlib (version 1.2.1 or greater)
- Tk/Tcl (version 8.5)

AshCalc may be compatible with previous versions of NumPy, SciPy and Matplotlib, although such compatibility has not been tested.

Set up instructions

If the requirements above are not met, the simplest way is to download the SciPy stack, which contains the latest versions of Python, NumPy, SciPy and Matplotlib bundled together in one package.

Various versions of the stack, along with installation instructions, can be found at <http://www.scipy.org/install.html>.

The **Setup** instructions only need to be performed once when first downloading AshCalc. Only the **Running the program** instructions need to be carried out when starting AshCalc every time after first downloading it.

Linux

On Linux based operating systems the Anaconda stack is recommended.

Setup

1) Installing Anaconda

Install Anaconda from <http://www.scipy.org/install.html>, following the instructions provided

2) Adding anaconda to the system path

Open a terminal and execute the command **export PATH=\$PATH:/dir/anaconda/bin** where **dir** is the location where Anaconda was installed
e.g. **export PATH=\$PATH:/home/user/anaconda/bin**

3) Creating a Python 3.3 environment

Open a bash terminal and execute the command

conda create -n py33 python=3.3 anaconda and follow the instructions.

Running the program

- 1) Activating the Python 3.3. environment

Open a bash terminal and execute the command **source activate py33**

- 2) Running AshCalc.py

Execute the command **python3 dir/ashcalc.py** where **dir** is the folder where AshCalc.tar was extracted to e.g. **python3 /Users/user/Documents/AshCalc/ashcalc.py**

Windows

For Windows operating systems the Pyzo stack is recommended.

Setup

- 1) Installing Pyzo

Install Pyzo from <http://www.scipy.org/install.html>, following the instructions provided.

- 2) Adding python to the "Path" system variable

Right click on "My Computer" (or just "Computer" in Windows 8) then click the "Properties", followed by "Advance System Settings" and then "Environment Variables". In this window under "System variables" find "Path" and click "Edit". To the start of the field add the directory where Pyzo was installed followed by a semicolon (for example "**C:\pyzo2013b;**")

Running AshCalc

- 1) Running ashcalc.py

Open the command line terminal. Execute the command **python dir/ashcalc.py** where **dir** is the folder where AshCalc.tar was extracted to e.g. **python /Documents/AshCalc/ashcalc.py**

Mac OS

On Mac OS X the Anaconda stack is recommended.

Setup

- 1) Installing Anaconda

Install Anaconda from <http://www.scipy.org/install.html>, following the instructions provided.

- 2) Creating a Python 3.3 environment

Open a bash terminal and execute the command **conda create -n py33 python=3.3 anaconda** and follow the instructions.

Running AshCalc

- 1) Activating the Python 3.3. environment

Open a bash terminal and execute the command **source activate py33**

- 2) Running ashcalc.py

Execute the command **python3 dir/ashcalc.py** where **dir** is the folder where AshCalc.tar was extracted to e.g. **python3 /Users/user/Documents/AshCalc/ashcalc.py**

Performing calculations

AshCalc provides three different models for tephra deposition, the exponential, the power-law and the Weibull model. All three work under the approximation that the deposit is circular and hence that the total volume of the deposit obeys the following equation

$$V = \int_0^{\infty} T(x) dA = 2 \int_0^{\infty} xT(x) dx$$

where A is the area of the isopach, x is either the square root of A or the distance to the vent and T is the thickness of the deposit at radius x. The three models differ on how deposit thickness, T(x), is related to x.

There are three steps needed to be taken in order to perform a calculation:

- 1) Choose a model
- 2) Choose what x represents
- 3) Enter the isopach data

Choosing a model

Exponential model

Theory

The exponential model assumes that tephra thickness obeys the following relationship:

$$T(x) = ce^{-mx}$$

Where parameter m, characterises the rate of decay in thickness as distance from the vent increases, and parameter c represents the initial thickness of the deposit at the vent.

As the rate of the decay may change as you move the further away from the vent, the exponential model may contain multiple segments, with each segment having its own values for parameters c and m.



Model

☒ Exponential model

☐ Power law model

☐ Weibull model

Number of segments: 2

Usage

- AshCalc allows between 1 and 5 exponential segments, with the number specified in the “number of segments” box.
- AshCalc finds the best fit for the exponential parameters using segmented least squares regression with the segment boundaries placed as to create a continuous curve. As a result of ensuring curve continuity, a poor fit may be observed when clearly non-exponentially distributed data is used.
- An n segment exponential model requires a minimum of 2n isopachs. Hence you cannot use a 3-segment exponential model for a 5 isopach data set but you can for one containing 6 unique isopachs.

Power law model

Theory

The power law model assumes that tephra thickness obeys the following relationship:

$$T(x) = cx^{-m}$$

Where parameter m , characterises the rate of decay in thickness as distance from the vent increases, and parameter c is a scaling factor.

As this function is infinite when integrated between 0 and infinity, proximal and distal limits for integral need to be chosen, beyond which the model is assumed to break down. The estimated volume for the power law model is then:

$$V = 2 \int_{\text{proximal limit}}^{\text{distal limit}} xT(x)dx$$

Model
☐ Exponential model
☒ Power law model
☐ Weibull model

Proximal limit of integration:
Distal limit of integration:

Usage

- The integration limits may be set in their respective boxes. The proximal limit must always be smaller than the distal limit.
- The estimated total volume for a tephra deposit is often very sensitive to the proximal limit of integration in particular and therefore some thought is needed as to the “best” values.
- The power law model requires a minimum of 2 isopachs to produce a valid answer.

Weibull model

Theory

The Weibull model assumes that tephra thickness obeys the following relationship:

$$T(x) = \theta \left(\frac{x}{\lambda}\right)^{k-2} e^{-\left(\frac{x}{\lambda}\right)^k}$$

Where parameters λ and k characterise the variable rate of decay in thickness as distance from the vent increases, and parameter θ is a scaling factor. For the optimal solution θ can be expressed in terms of λ and k (see accompanying paper) and therefore only these two parameters need to be determined.

As $T(x)$ cannot be linearised, an optimisation algorithm is needed to find estimates for the values of λ and k . AshCalc uses a random restart stochastic hill-climbing optimisation algorithm. What this means in practice is that for each run it picks initial random values for k and λ . During the run it performs iterations, changing the values for λ and k slightly and seeing if this produces a better solution. If the changes result in a better solution it accepts the changes and then continues to try and find another improvement. After a predefined number of iterations the run terminates and the best values found in that run are returned. A number of these runs are performed and the best overall result is therefore returned.

The screenshot shows the 'Model' configuration window of AshCalc. It contains three radio buttons for selecting a model: 'Exponential model', 'Power law model', and 'Weibull model'. The 'Weibull model' is currently selected. To the right of these buttons are several input fields for configuring the optimization process: 'Number of runs' (set to 10), 'Iterations per run' (set to 1000), 'Estimated time (s)' (set to 11), ' λ lower bound' (set to 0.0), ' λ upper bound' (set to 1000), 'k lower bound' (set to 0.0), and 'k upper bound' (set to 10.0).

Usage

- As the process of estimating values for λ and k is inherently random, there is no guarantee that any particular calculation will provide a suitable answer (although with a suitable number of runs and iterations the probability is high). If an unsatisfactory fit is found it is recommended the calculation is performed again, taking into account suggestions below.
- AshCalc allows you to specify the number of runs and the number of iterations per run that the hill-climbing algorithm performs. Increasing either the number of runs or the iterations per run increases the probability that a good solution will be found.
- If poor fits are being found, it is recommended that the number of runs is increased before the number of iterations per run.
- Increasing the number of isopachs, the number of runs or the number of iterations per run will result in a corresponding increase in calculation time.
- An estimate of time required to make the calculation is provided. It should be stressed that although this estimate is a result of performance tests on the current platform that AshCalc

is being run on, this figure should be viewed as an order of magnitude estimate only. Actual calculation time may vary, particularly when a significant number of other tasks are being performed in the background.

- AshCalc requires bounds for λ and k to be specified, restricting the values that they may take. **Restricting the bounds inappropriately will result in consistently poor fits for the data.**
- It is recommended that initially loose bounds are provided (e.g. $0 \leq \lambda \leq 1000$, $0 \leq k \leq 10$). When a reasonable fit for the data is found, the bounds can then be further tightened. For example if it was found that $\lambda = 2.63$ and $k = 0.849$ provided a reasonable fit, the bounds could then be tightened to $1 \leq \lambda \leq 5$, $0 \leq k \leq 2$.
- Investigating the error surface in the results section can often be extremely useful in setting optimal bounds.
- An in-depth analysis for setting initial bounds can be found in Costanza Bonnadonna's paper "*Plume height, volume, and classification of explosive volcanic eruptions based on the Weibull function*" published in the Bulletin of Volcanology (2013).
- The Weibull model requires a minimum of two isopachs to produce a valid answer.

Choosing what x represents

In the literature x is usually the square root of isopach area in the equation:

$$V = \int_0^{\infty} T(x) dA = 2 \int_0^{\infty} x T(x) dx$$

The square root of area is closely related to the distance from the vent:

$$\sqrt{Area} = Vent\ distance \sqrt{\pi}$$

Using distance from the vent as an alternative representation of x greatly increases the intuitive meanings of several parameters. An example of this, using x as vent distance, a proximal limit for integration in the power law model of 1.5 actually represents 1.5km from the vent, rather than 2.659km when square root of area is used.

In order to make calculations more intuitive AshCalc provides the option to switch between both representations.

Options

For x in $\int xT(x)dx$:

☒ use vent distance

☐ use \sqrt{Area}

The table below shows how the parameters are related for the two different representations:

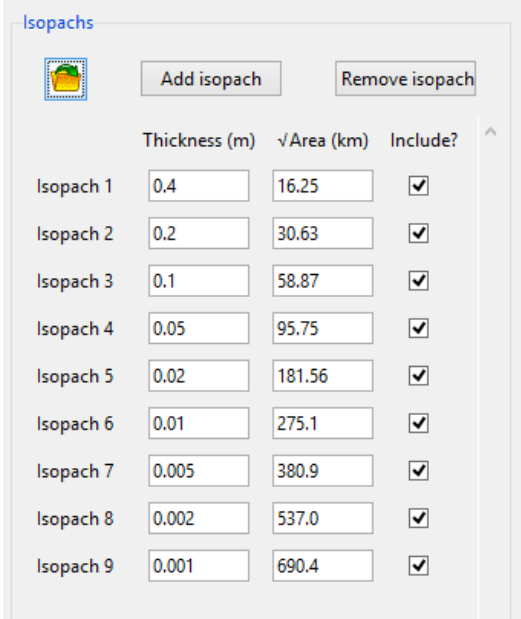
Model	Variable	\sqrt{Area}	Vent distance
Power law	Proximal limit of integration	$prox_A$	$prox_V = \frac{prox_A}{\sqrt{\pi}}$
	Distal limit of integration	$dist_A$	$dist_V = \frac{dist_A}{\sqrt{\pi}}$
	Parameter m	m_A	$m_V = m_A$
	Parameter c	c_A	$c_V = c_A \sqrt{\pi}^{m_A}$
Weibull	Wei parameter λ	λ_A	$\lambda_V = \lambda_A \sqrt{\pi}$
	Wei parameter k	k_A	$k_V = k_A$
	Wei parameter θ	θ_A	$\theta_V = \theta_A$
Exponential	Parameter m	m_A	$m_V = m_A \sqrt{\pi}$
	Parameter c	c_A	$c_V = c_A$

Entering isopach data

AshCalc requires isopachs to be entered as a pair of values, the thickness of the isopach (measured in metres) and the square root of the area the isopach covers (measured in kilometres).

AshCalc can handle an arbitrary number of isopachs, with additional isopachs added at the end of the list by using the “Add” button. Similarly the “Remove” button can be used to delete the last isopach in the list. There must be a minimum of 2 isopachs in the list.

If you want to temporarily remove an isopach, (for example to see the effects of an outlier), you can uncheck the tick box in the “Include?” column. This will remove it from consideration during the calculations phase. It can be included again by simply rechecking the tick box.



	Thickness (m)	√Area (km)	Include?
Isopach 1	0.4	16.25	<input checked="" type="checkbox"/>
Isopach 2	0.2	30.63	<input checked="" type="checkbox"/>
Isopach 3	0.1	58.87	<input checked="" type="checkbox"/>
Isopach 4	0.05	95.75	<input checked="" type="checkbox"/>
Isopach 5	0.02	181.56	<input checked="" type="checkbox"/>
Isopach 6	0.01	275.1	<input checked="" type="checkbox"/>
Isopach 7	0.005	380.9	<input checked="" type="checkbox"/>
Isopach 8	0.002	537.0	<input checked="" type="checkbox"/>
Isopach 9	0.001	690.4	<input checked="" type="checkbox"/>

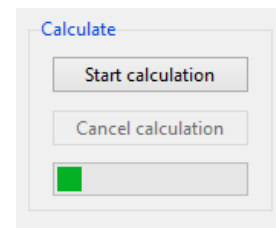
It is also possible to load isopach data from a CSV (comma separated value) file, using the load button in the top left. The file data must be in the following format:

```
thickness1,sqrtArea1
thickness2,sqrtArea2
thickness3,sqrtArea3
....
```

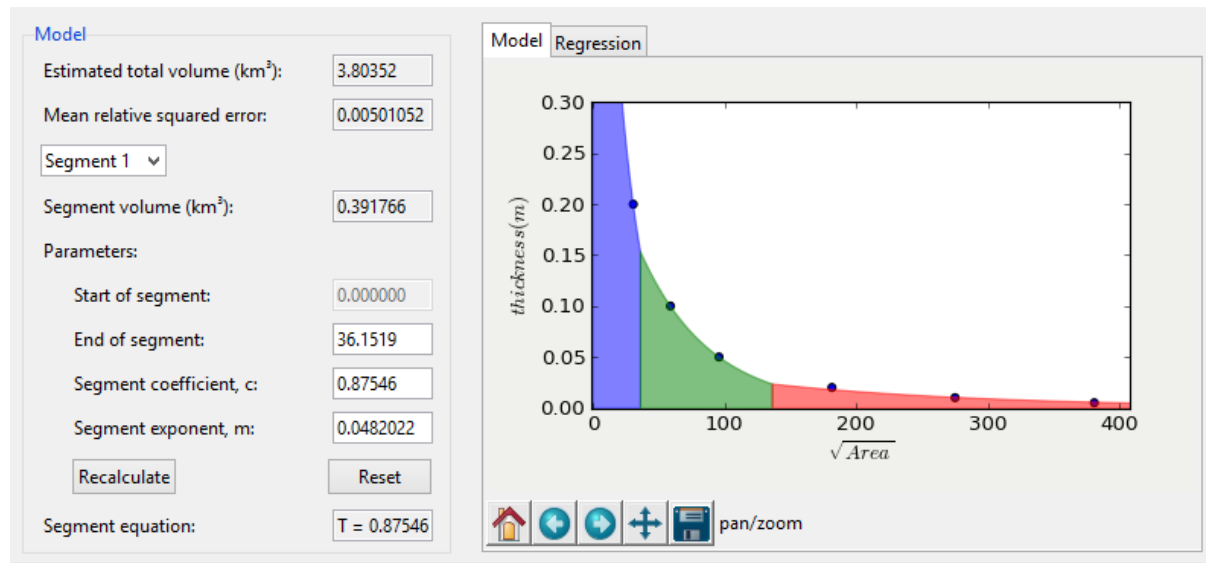
Results

Once the isopach data has been entered and the model and its corresponding options have been chosen, press the “Start calculation” button in the top left hand corner.

The calculation can be cancelled prematurely by clicking the “Cancel calculation” button.



Once the calculation is complete the results for the model are displayed in the bottom right hand side of the display.



Numerical results

The following fields exist for each of the three models:

- **Estimated total volume** – the total volume of the tephra deposit as predicted by the model with the current parameters
- **Mean relative squared error** – a measure of how well the model fits the data. It represents the average relative difference in thickness between the model and a data point.

$$MRSE = \frac{1}{n} \sum_{x \in data} \left(\left(\frac{T_{obs}(x) - T(x)}{T_{obs}(x)} \right)^2 \right)$$

The closer the value is to zero the better the fit. It can be used to as a method to compare the accuracy of different models. For the comparisons to be meaningful however it is necessary for the isopach data to be identical for the different models.

- **Parameters** –The parameter fields are editable and allow the user to tweak parameters themselves. The changes will be reflected in the other fields and on the graphs to the right when the “Calculate” button is pressed. The “Reset” button can be used to return the parameters to their original values.

- **Equation** – A textual representation of the simplest form of current equation for thickness. The expression “ x^y ” represents x raised to the power of y.

In the exponential model segment details are viewable individually. The current segment can be chosen using the drop down list. The additional fields for each segment are:

- **Segment volume** – the volume of tephra deposit within the bounds of each segment.
- **Start of segment** – the x coordinate at which the current segment’s equation becomes valid. The first segment will always start at zero (i.e. at the vent)
- **End of segment** – the x coordinate at which the current segment’s equation is no longer valid. The last segment will always end at infinity.

Graphical results

For all three models the graph under the “Model” tab shows x plotted against thickness overlaid with the isopach data points.

The exponential and power law models also have a “Regression” tab which shows the results of the least squares regression fit.

Navigating graphs

The graphs provided by AshCalc are interactive and provide a variety of tools as shown below:

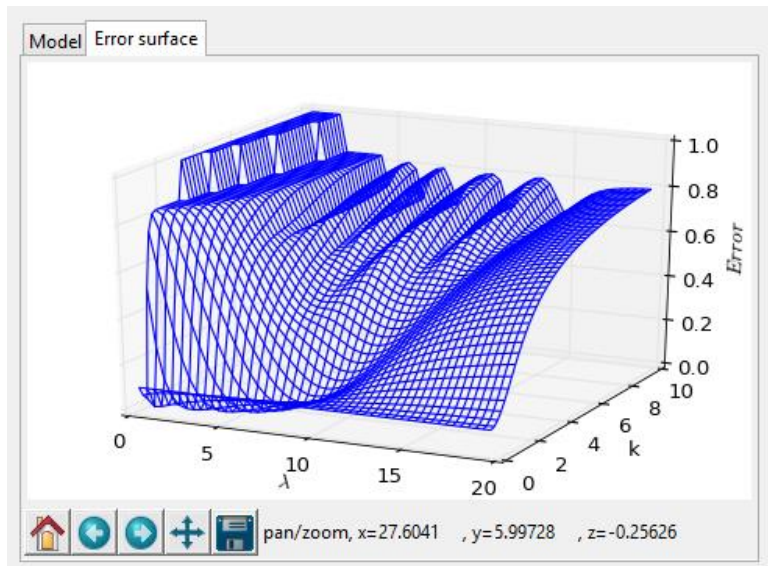


From left to right:

- Home button – Resets the view of the graph to the original view displayed
- Back button – Returns the graph to the previous view
- Forward button – Returns the graph to the subsequent view
- Navigation button – Allows the control of the view of the graph. Left clicking on the graph and dragging will result in the view panning across the graph. Right clicking on the graph and dragging will result in changing the magnification of the graph.
- Save button – Opens a file dialog where you can save an image of the graph in a variety of image formats including .jpeg, .png and .pdf

Model error surfaces

For the power law and the Weibull model, AshCalc can produce a 3 dimensional plot of the error surface. The two parameters of the models are on the x and y axes, whilst the error for the fit is plotted on the z-axis. The smaller the error, the better the curve shown on the “Model” tab will fit the data.



For example in the figure above, it can be seen that curves in which $0 \leq k \leq 2$ will provide a much better fit than those in which $4 \leq k \leq 10$. This feature is particularly of use when trying to set tight bounds for λ and k in the Weibull model's options panel.

Error surface

Lower limit (λ):	<input type="text" value="0.0"/>	Upper limit (λ):	<input type="text" value="20.0"/>	Resolution:	<input type="text" value="50"/>
Lower limit (k):	<input type="text" value="0.0"/>	Upper limit (k):	<input type="text" value="10.0"/>	<input type="button" value="Calculate error surface"/>	

The region shown on the graph can be controlled in the error surface panel. The resolution field affects how often the error function is sampled. For example with a resolution of 50 the error function is sampled on a 50x50 grid within the specified region.

Increasing the resolution excessively will result in poor response times when panning or zooming with the graph. A resolution value of 50 is usually a good compromise between detail and response times on a middle of the range computer.

The multi-segment exponential model has more than two parameters and hence its error surface cannot be plotted in 3 dimensions.