

## ***Tephra2***

***Tephra2 uses the advection-diffusion equation to calculate tephra accumulation at locations about a volcano based on a pre-defined set of eruptive conditions.***

***This module explains how to use TEPHRA2***

***Photo by P.C. LaFemina***

***Authors: Laura Connor, Chuck Connor  
University of South Florida, Tampa***

***Tephra2 is open source software***

***Manual by: Leah Courtland***

***This module presents background information on the tephra2 code, explains how to implement the program both through the rappture gui and at the command line, and provides a number of examples which demonstrate the resourcefulness of the model.***

***Slide 3 gives some background on tephra hazard models***

***Slides 4 - 5 give the short explanation of how Tephra2 works***

***Slide 5 over view of the steps and assumptions of the model***

***Slides 6 - 12 explain how to use the Tephra2 GUI***

***Slide 13 - 17 explain how to run Tephra2 at the command line, go over the input files***

***Slide 18 explains the code output***

***Slide 19- 26 a more thorough explanation of how Tephra2 works***

***Slide 27 example eruption: Cerro Negro, 1992***

***Slide 28 example eruption: Irazu, 1968***

***Slide 29 example eruption: Pululagua, 2350 BP***

***Slides 30 – 31 give a more mathematical explanation of the advection-diffusion equation***

***Slide 32 provides additional references.***

***Tephra sedimentation models are used to forecast the accumulation of ejected volcanic material across a region. We need tephra fallout models that we can trust in order to make reliable forecasts crucial to hazard assessment.***

### HAZARD MODELS

***Numerical models used for tephra hazard assessment (Hazard Models) typically result from the combination and integration of different theories and modeling approaches depending on the specific eruptive scenario and mitigation program required. They can be grouped within two main categories: particle-tracking models and advection-diffusion models. Particle-tracking models are Eulerian or Lagrangian models that can forecast volcanic-cloud position at specific times and space. They are mainly used for aviation-safety purposes. Advection-diffusion models are Eulerian models that describe the solution of the equations of particle diffusion, transport, and sedimentation and can forecast tephra accumulation on the ground relative to a particle-release source. These models are mainly used for civil protection purposes, such as giving public warnings and planning mitigation measures.***

## Tephra2: The Inputs:

**Eruption Parameters**      **Volcano Location and Height**  
**Mass Erupted**  
**Particle Size Distribution**  
**Plume Height**  
**Integration Steps**

**Particle Parameters:**      **Fall Time Threshold**  
**Diffusion Coefficient**  
**Lithic and Pumice Densities**  
**Integration Steps**  
**Plume Dispersion Model**

**Atmospheric Parameters:**      **Atmosphere Levels**  
**Wind Speed**  
**Wind Directions**

**Fallout (Grid) Parameters:**      **Grid Pt Locations**  
**Elevation**

## Tephra2: The Outputs:

**Advection  
Diffusion**

**Mass per  
Unit Area**

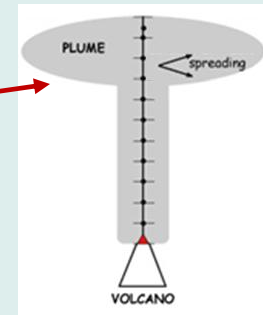
**WT.% Phi  
Size**

## Advection Diffusion

Double Integration at each grid location:

$$M(x, y) = \sum_{i=1}^H \sum_{j=\phi_{\min}}^{\phi_{\max}} M_{i,j}^0 f_{i,j}(x, y)$$

1. For each step up the eruption column (i) [vent to max column height (H)]
2. For each particle size (j) [max  $\phi$  to min  $\phi$ ]
3. Calculate probability of particle size release at each step
4. Calculate particle fall time from a point release source
5. Modify by column diffusion
6. Modify by atmospheric diffusion
7. Fall through a layered atmosphere
8. Accumulate mass (M) at each grid location (x,y)



## ***Running Tephra2: the GUI: Quick and Dirty Version***

***Once you have launched the Tephra2 Tool, simply:***

***Load a wind file  
Load a grid file  
Load a config file OR input config values into the GUI  
Press 'Simulate'***

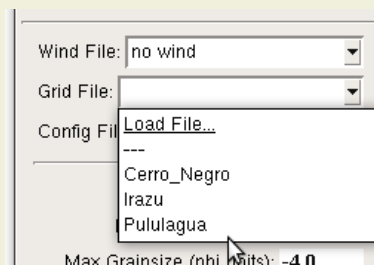


# Running Tephra2 : GUI : Input

To run Tephra2 from the vhub site, launch the Tephra 2 tool. In a moment, the graphical interface will appear:

To load the wind, grid, or configuration files, select an option from the drop down menu:

Here the user is selecting the grid for Pululagua. Pululagua 2350BC is one of the example eruptions.



Loading a configuration file is optional as each of the values defined in the configuration file may be defined via the graphical interface. Simply select any of the fields to change the value within.

It is important to note that the values shown will not change if one loads a configuration file from their local computer. However, the values in the loaded config file, and not those shown on the screen, will be used in the calculation.

A screenshot of the Tephra2 GUI. The 'File' menu is open, showing the 'Input' tab selected. The 'Input' tab contains several dropdown menus for 'Wind File' (set to 'no wind'), 'Grid File', and 'Config File (Optional)'. Below these are various configuration fields, all set to 0: 'Plume Height (m)', 'Eruption Mass (kg)', 'Max Grainsize (phi units)', 'Min Grainsize (phi units)', 'Median Grainsize (phi units)', 'STD Grainsize (phi units)', 'Vent Easting (UTM)', 'Vent Northing (UTM)', 'Vent Elevation (asl)', 'Eddy Constant', 'Diffusion Coefficient (m^2/s)', 'Fall Time Threshold (s)', 'Lithic Density (kg/m^3)', 'Pumice Density (kg/m^3)', 'Column Steps', 'Plume Model', and 'Plume Ratio'. A 'Simulate >' button is at the bottom right. A red arrow points from the 'Grid File' dropdown in the 'Input' tab to the 'Grid File' dropdown in the 'File' menu. A black arrow points from the 'Config File (Optional)' dropdown in the 'Input' tab to the 'Config File (Optional)' dropdown in the 'File' menu. A black arrow points from the 'Simulate >' button in the 'Input' tab to the 'Simulate >' button in the 'File' menu.

Configuration values may either be set here or loaded from file

# Running Tephra2 : GUI : Input

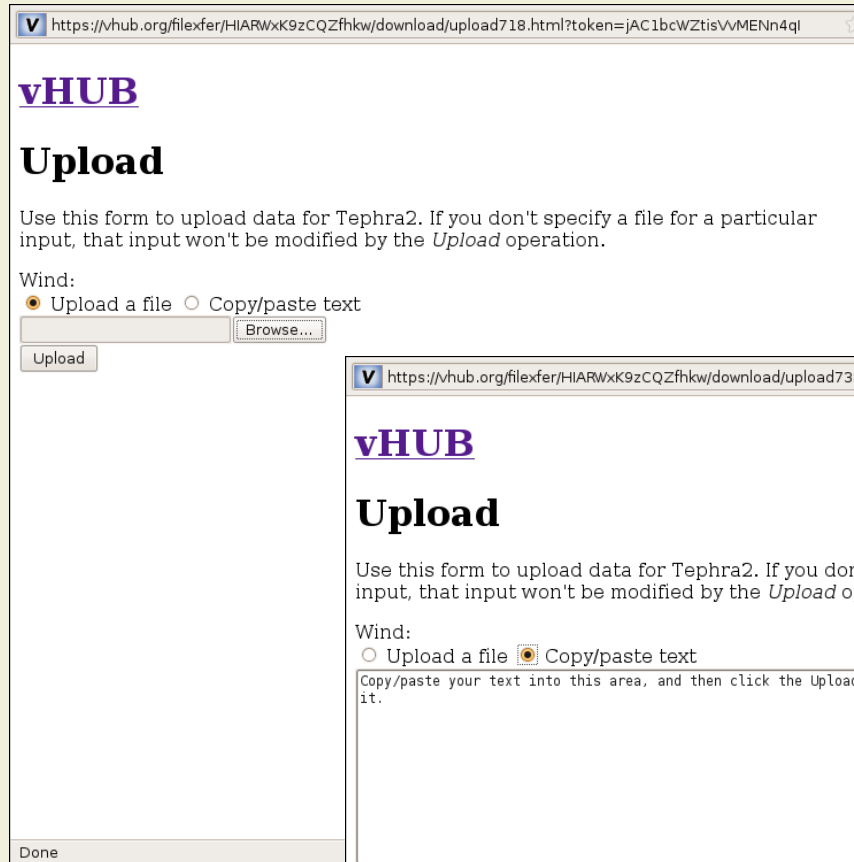
**To upload a file from your home computer, select the upload option. Make sure pop-ups are enabled.**

**The following window will appear:**

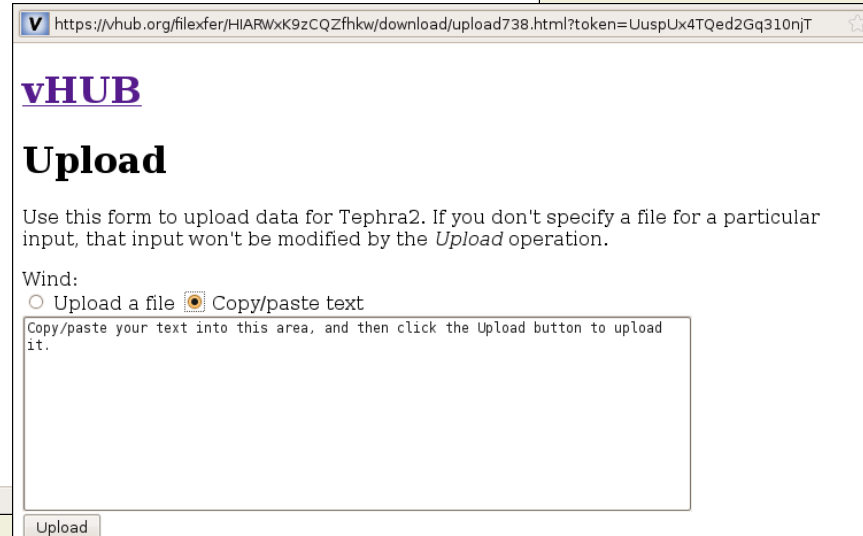
**To load a file, select 'Browse' and indicate the file you wish to upload.**

**Alternatively, you may type or copy and paste information into the browser window. This option might be useful if you wished to know the amount of tephra accumulating at only one or a few points.**

**The necessary file format is described further on slide 22.**



The screenshot shows a browser window with the URL <https://vhub.org/filexfer/HIARWxK9zCQZfhkw/download/upload718.html?token=jAC1bcWZtisVvMENn4qI>. The page title is "vHUB" and the main heading is "Upload". Below the heading, there is a paragraph of text: "Use this form to upload data for Tephra2. If you don't specify a file for a particular input, that input won't be modified by the Upload operation." Underneath, there is a label "Wind:" followed by two radio buttons: "Upload a file" (which is selected) and "Copy/paste text". Below the radio buttons is a text input field with a "Browse..." button to its right. At the bottom of the form is an "Upload" button. The status bar at the bottom of the browser window shows "Done".



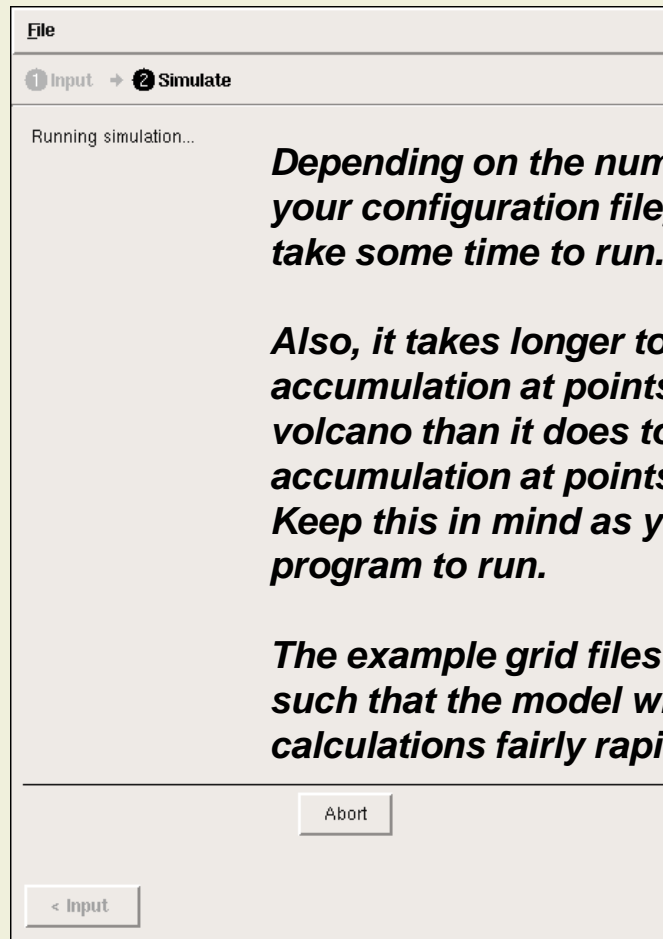
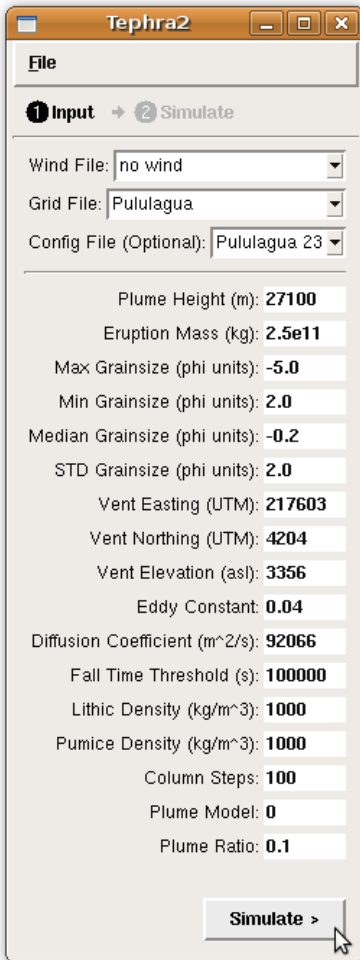
The screenshot shows a browser window with the URL <https://vhub.org/filexfer/HIARWxK9zCQZfhkw/download/upload738.html?token=UuspUx4TQed2Gq310njT>. The page title is "vHUB" and the main heading is "Upload". Below the heading, there is a paragraph of text: "Use this form to upload data for Tephra2. If you don't specify a file for a particular input, that input won't be modified by the Upload operation." Underneath, there is a label "Wind:" followed by two radio buttons: "Upload a file" and "Copy/paste text" (which is selected). Below the radio buttons is a text area with the instruction: "Copy/paste your text into this area, and then click the Upload button to upload it." At the bottom of the form is an "Upload" button.



# Running Tephra2 : GUI : Simulation

**Once all of the parameters have been specified, hit 'simulate' to run the model.**

**The following screen will appear:**



**Depending on the number of points in your configuration file, Tephra2 may take some time to run.**

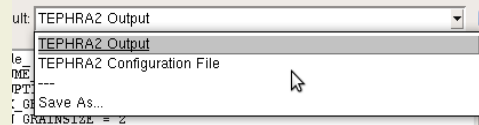
**Also, it takes longer to calculate tephra accumulation at points far from the volcano than it does to calculate accumulation at points near the vent. Keep this in mind as you wait for the program to run.**

**The example grid files were chosen such that the model will complete all calculations fairly rapidly.**

# Running Tephra2 : GUI : Output

Tephra2 currently has two outputs, shown below:

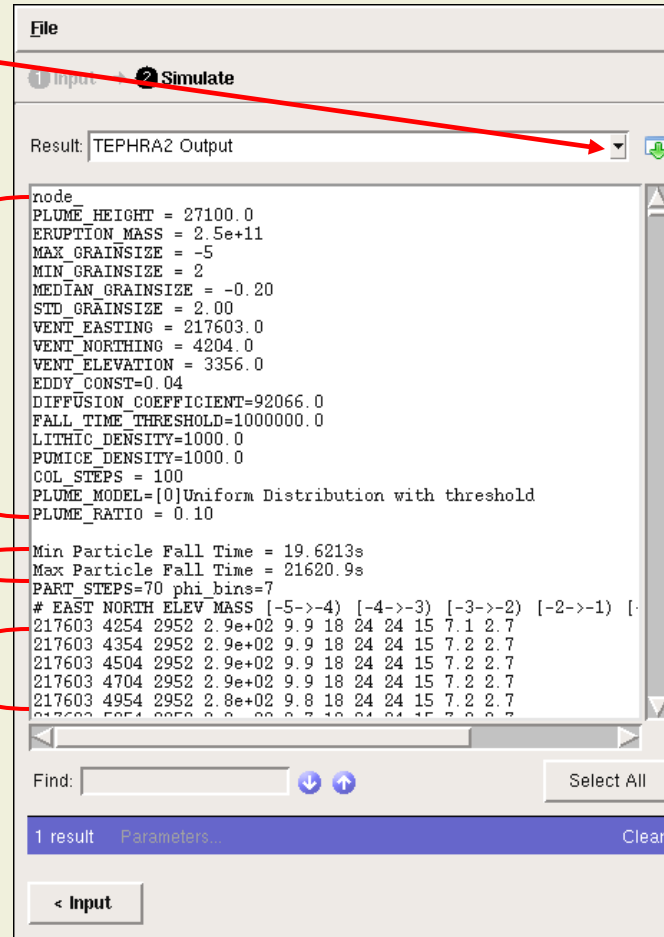
To toggle between the two outputs, select here



Parameters used in model calculations.

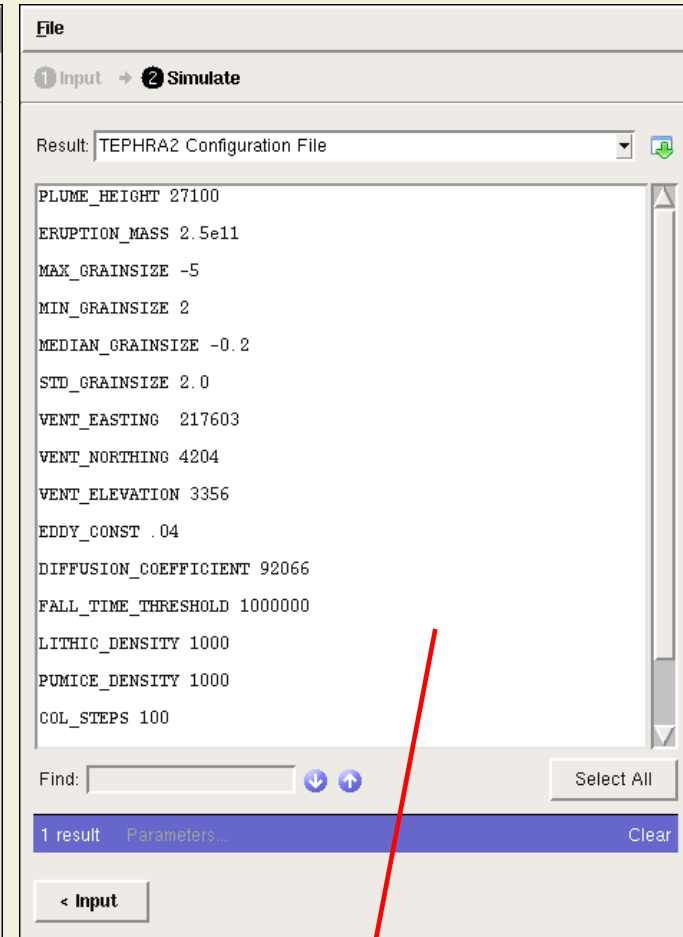
Min and Max particle fall times

Code output:  
Easting, Northing,  
Elevation, Mass per  
unit area, percent  
phi. . .



```
node
PLUME_HEIGHT = 27100.0
ERUPTION_MASS = 2.5e+11
MAX_GRAINSIZE = -5
MIN_GRAINSIZE = 2
MEDIAN_GRAINSIZE = -0.20
STD_GRAINSIZE = 2.00
VENT_EASTING = 217603.0
VENT_NORTHING = 4204.0
VENT_ELEVATION = 3356.0
EDDY_CONST=0.04
DIFFUSION_COEFFICIENT=92066.0
FALL_TIME_THRESHOLD=1000000.0
LITHIC_DENSITY=1000.0
PUMICE_DENSITY=1000.0
COL_STEPS = 100
PLUME_MODEL=[0]Uniform Distribution with threshold
PLUME_RATIO = 0.10

Min Particle Fall Time = 19.6213s
Max Particle Fall Time = 21620.9s
PART_STEPS=70 phi_bins=7
# EAST NORTH ELEV MASS [-5->-4) [-4->-3) [-3->-2) [-2->-1) [-1->0)
217603 4254 2952 2.9e+02 9.9 18 24 24 15 7.1 2.7
217603 4354 2952 2.9e+02 9.9 18 24 24 15 7.2 2.7
217603 4504 2952 2.9e+02 9.9 18 24 24 15 7.2 2.7
217603 4704 2952 2.9e+02 9.9 18 24 24 15 7.2 2.7
217603 4954 2952 2.8e+02 9.8 18 24 24 15 7.2 2.7
```

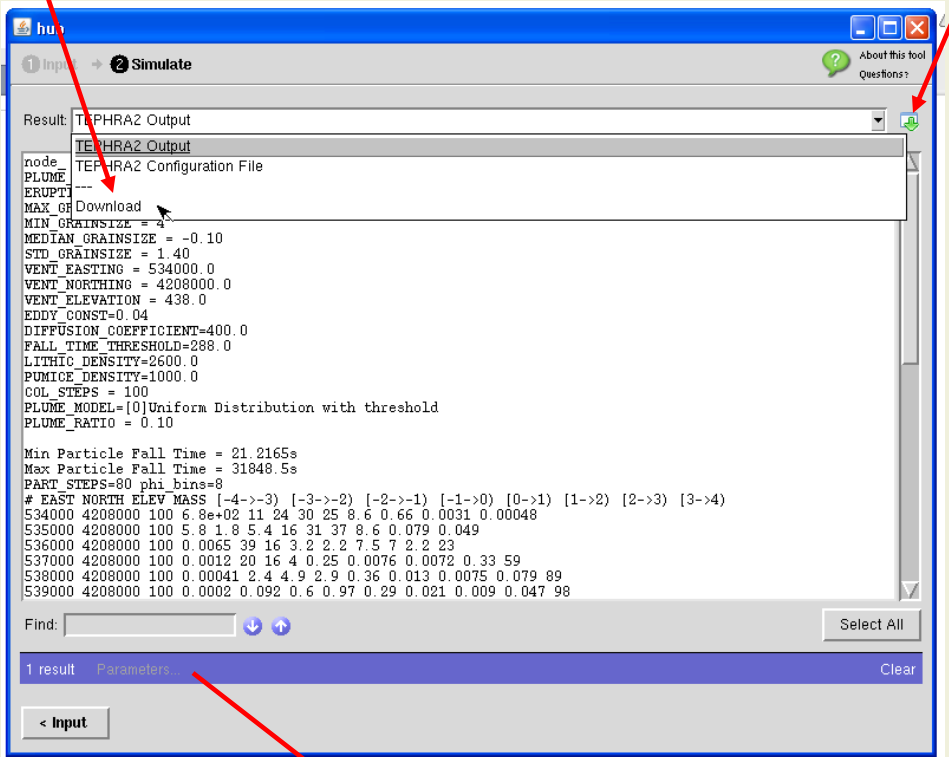


```
PLUME_HEIGHT 27100
ERUPTION_MASS 2.5e11
MAX_GRAINSIZE -5
MIN_GRAINSIZE 2
MEDIAN_GRAINSIZE -0.2
STD_GRAINSIZE 2.0
VENT_EASTING 217603
VENT_NORTHING 4204
VENT_ELEVATION 3356
EDDY_CONST .04
DIFFUSION_COEFFICIENT 92066
FALL_TIME_THRESHOLD 1000000
LITHIC_DENSITY 1000
PUMICE_DENSITY 1000
COL_STEPS 100
```

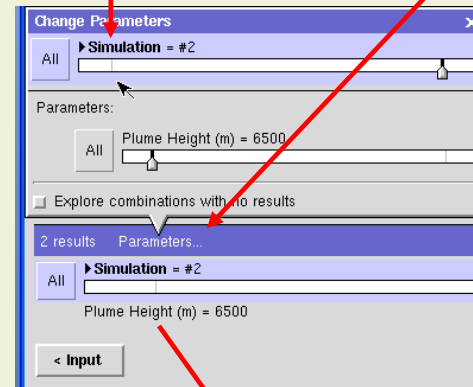
The 'Configuration File' output provides the configuration parameters in the format required by the program. This output may be saved and subsequently uploaded into the GUI. This may be useful if parameters were input by hand.

# Running Tephra2 : GUI : Output

To save the output, either select 'Download' from the drop down menu, or click on the little arrow to the left of this menu.



It is possible to toggle between the results of multiple runs by selecting 'Parameters. . .' and then clicking on the grey line indicating the simulation number.



Indicates the number of times the code has been run and which output we are currently looking at. The parameters option will indicate which parameters changed between runs.

Indicates that the parameter 'Plume Height' was changed between simulations 1 & 2.

To return to the input screen and upload parameters for a second run, select the '< Input' button.

Note that the parameters of the previous run will be saved in the GUI.

# Running Tephra2: the GUI

## Changing the configuration file:

**The easiest way to change the configuration file is to open the file in a text reader, type in any changes, and save the file. Simply upload the updated file into the GUI in order run the program with the revised configuration.**

**Another option is to input all of your configuration file information into the GUI fields, changing any parameters that need refreshing. Run the GUI to determine results. To change a parameter at this point, return to the input screen and make the necessary changes.**

**File**

1 Input → 2 Simulate

Wind File: no wind

Grid File: Uploaded data

Config File (Optional): Previous Run

---

Plume Height (m): 27100

Eruption Mass (kg): 2.5e11

Max Grainsize (phi units): -5.0

Min Grainsize (phi units): 2.0

Median Grainsize (phi units): -0.2

STD Grainsize (phi units): 2.0

Vent Easting (UTM): 217603

Vent Northing (UTM): 4204

Vent Elevation (asl): 3356

Eddy Constant: 0.04

Diffusion Coefficient (m<sup>2</sup>/s): 92066

Fall Time Threshold (s): 100000

Lithic Density (kg/m<sup>3</sup>): 1000

Pumice Density (kg/m<sup>3</sup>): 1000

Column Steps: 100

Plume Model: 0

Plume Ratio: 0.1

Simulate >

## ***Running Tephra2: Command Line***

***Once Tephra2 has been downloaded and installed, simple enter the following into the command line:***

```
./tephra2 config.file grid.file wind.file > tephra2_output.txt
```

## Running Tephra 2: Command Line: Input

***TEPHRA2 requires three input files: a configuration file, a grid file, and a wind file.***

***The configuration file is a text file consisting of KEY-NAME VALUE pairs, one pair per line, separated by a space. The KEY-NAME must not be changed. Only the VALUE should be changed by the user.***

***Comment lines in the configuration file must begin with the symbol: #***

***The grid file is a text file consisting of three values per line, values separated by a space. These 3-tuples are determined by the user and represent locations around the volcano where mass per unit area values of tephra are calculated by the program. Easting and northing are specified in meters (usually UTM coordinates with respect to a particular zone) and the elevation is specified as meters above sea level. Zero can be specified for the elevation, meaning at sea level. If elevations are not known, a consistent average elevation value can also be used. The format being:***

***EASTING(m) NORTHING(m) ELEVATION(m)***

***The wind file is a text file consisting of three values per line (again separated by spaces), height above sea level, wind speed at this height, and direction the wind is blowing toward at this height. The file format is:***

***HEIGHT(m) WINDSPEED(m/s) DIRECTION(+/- degrees)***

### Configuration File

The **KEY-NAMES** for the configuration file are as follows (continued on next slide):

**PLUME\_HEIGHT** Maximum height of tephra column (meters above sea level)

**ERUPTION\_MASS** Total mass of tephra erupted from the volcano (kg)

**MAX\_GRAINSIZE** Maximum particle size of tephra erupted from the volcano (phi units)

**MIN\_GRAINSIZE** Minimum particle size of tephra erupted from volcano (phi units)

**MEDIAN\_GRAINSIZE** Median particle size erupted from volcano (phi units)  
(the mean of a Gaussian distribution)

**STD\_GRAINSIZE** Standard deviation of particle size of tephra erupted from volcano (phi units)

**VENT\_EASTING** Easting location of vent (UTM coordinates)

**VENT\_NORTHING** Northing location of vent (UTM coordinates)

**VENT\_ELEVATION** Elevation of vent (UTM coordinates)



### Configuration File:

*The KEY-NAMES for the configuration file (continued):*

**EDDY\_CONST** *Describes atmospheric diffusion; Fickian diffusion parameter*

**DIFFUSION\_COEFFICIENT** *Term describing the advection and diffusion of particles through the atmosphere*

**FALL\_TIME\_THRESHOLD** *The maximum time limit by which particles need to land*

**LITHIC\_DENSITY** *Density of small particles (kg/m<sup>3</sup>)*

**PUMICE\_DENSITY** *Density of large particles (kg/m<sup>3</sup>)*

**COL\_STEPS** *Number of segments into which to discretize the eruption column*

**PLUME\_MODEL** *Describes distribution of particles in the eruption column based on mass; 0 = a well-mixed plume*

**PLUME\_RATIO** *Describes where in the column the majority of mass resides.*

*1: tephra emitted only from the top of the column, 0: released from every height.*

## Running Tephra 2: Command Line: Input

### *Wind File*

<i>HEIGHT (m)</i>	<i>WINDSPEED (m/s)</i>	<i>DIRECTION (+/- degrees)</i>
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### *Grid File*

<i>EASTING (m)</i>	<i>NORTHING (m)</i>	<i>ELEVATION (m)</i>
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*For best results, the elevation should be equal for all points on the grid. Once the calculation is complete, output may be draped over topography if desired.*

## Running Tephra 2: Output

**Mass/area**

**WT.% Phi Size**

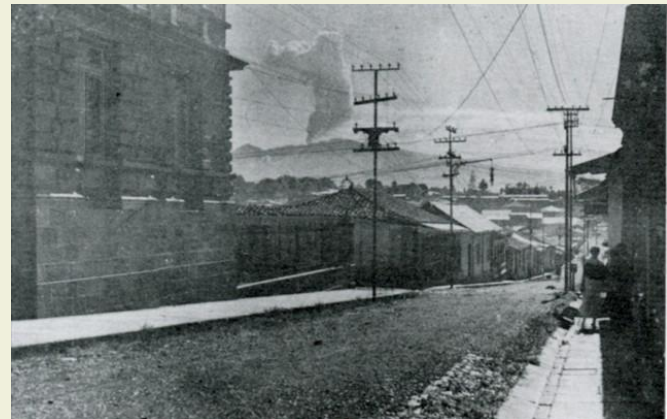
***TEPHRA2 outputs mass per unit area and the associated weight percent of each phi size at each grid point specified in the input grid file. The range of phi sizes is specified in the input configuration file.***

***The file format is:***

***EASTING NORTHING ELEVATION MASS WT.-%-MIN\_PHI ..... WT.-%-MAX\_PHI***

***An eruption plume above the summit crater of Irazú is seen here in 1917 from the national theater in San José. All summit craters were reported to be smoking on September 27, 1917, after which activity steadily intensified.***

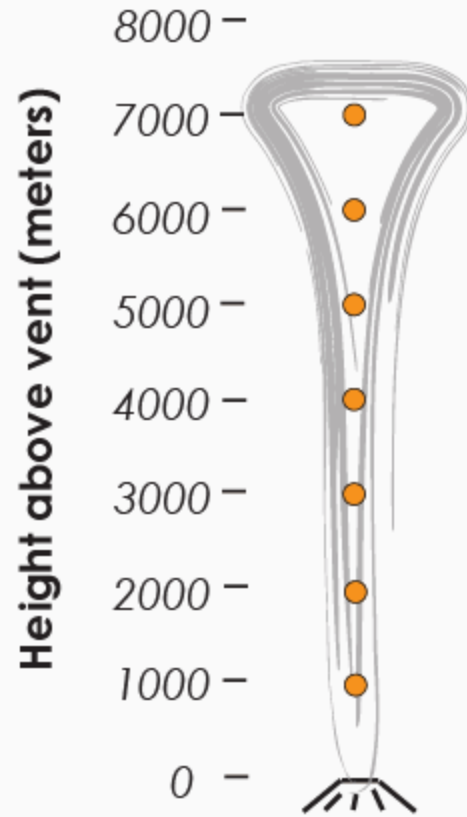
***Anonymous photo. Taken from the GVP Website:  
<http://www.volcano.si.edu/world/volcano.cfm?vnum=1405-06=&volpage=photos>***



## Eruption Column

**1. A vertical eruption column is assumed to extend above the vent.**

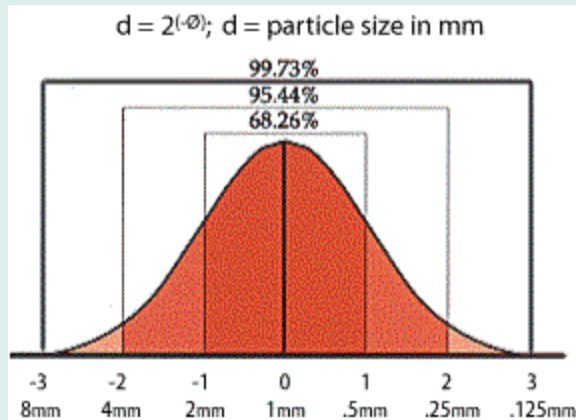
**The column is discretized and particles fall from each column height**



*Cerro Negro Volcano, Nicaragua*

## Grain Size Distribution

2. The total grain size distribution is estimated for the eruption, assuming a normal distribution in phi units :



The grain size distribution of a deposit defines the relative amount of particles present in the deposit, sorted according to particle size.

A typical tephra deposit will have a median grain size between -1 and 1 phi (0.5 – 1.0 mm).



Image shows three tephra fall units, each exhibiting a different grain size distribution

## ***Eruption Column***

***3. The total tephra mass is distributed vertically in the eruption column based on a probability density function for mass as a function of height. For example, for an umbrella cloud, mass might be equally distributed in the upper 20% of the total column height.***



***A Plinian eruption column at Lascar volcano, Chile, in 1993.***



## Particle Fall Time

4. Calculate the total fall time of a particle from the point of release in the eruption column to the ground surface. The particle fall time depends on particle properties (density, diameter) and atmospheric density. Settling velocity is determined assuming spherical particles and accounting for the variation in particle Reynolds Number and atmospheric density.



Gravity and air resistance help to govern the fall of particles from the atmosphere. Most particles reach a terminal velocity after a few seconds.

The fall of small particles is dominated by air resistance while the fall of large particles is dominated by gravity.

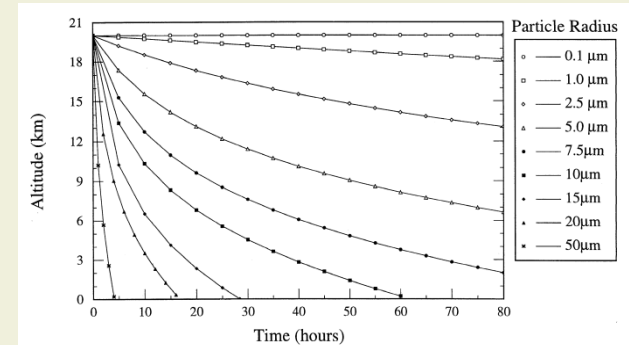


Figure 3. Fall distances for spherical, 0.1–50  $\mu\text{m}$  radius, volcanic ash particles as a function of time. The local viscosity was determined from a radiosonde temperature profile collected at Veracruz, Mexico (380 km NW of El Chichón) on April 5 at 0000 UT.

Schneider et al., 1999, *J Geophys Res* 104  
4037–4050



## Eruption Column

**5. Eruption column radius increases with height and this affects deposition. Particles are more spread out than they would be if they were released from a vertical line source. This effect is accounted for by increasing the diffusion time as a function of height in the column.**

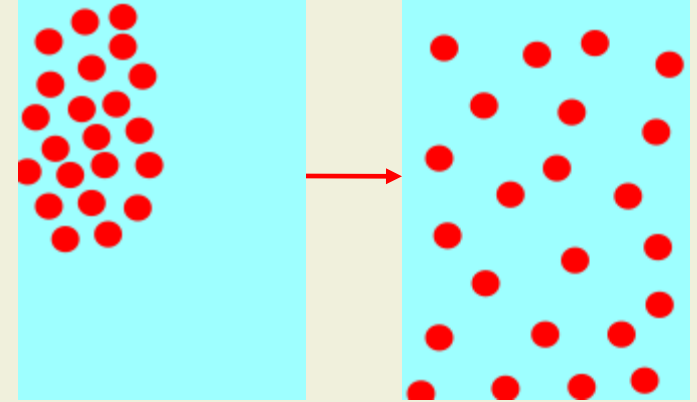
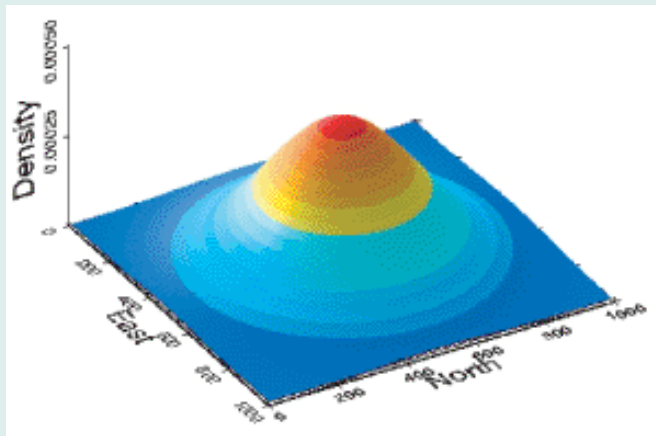


**A dense ash plume pours from the summit crater of Irazú volcano on April 14, 1964, about midway through a major explosive eruption that began in 1963.**

*Photo by USGS 1964. Obtained from GVP Website.*

## Particle Diffusion

6. Diffusion of particles in the atmosphere is estimated using a bivariate Gaussian probability density function to approximate turbulence. The scale of diffusion (described by the diffusion law) depends on total particle fall time, which depends on particle size, release height, and elevation of the terrain the particle impacts.



### Atmospheric Affects

**7. Particles fall through a stratified atmosphere. Wind speed and direction change between layers, based on REANALYSIS or locally collected wind data.**



### Tephra Accumulation

8. Total tephra accumulation is estimated as mass per unit area; most hazard results from excessive mass loading. The isomass is contoured over a region about the volcano. Mass per unit area can be easily converted into thickness knowing the deposit bulk density (i.e.  $\text{thickness (m)} = \text{kgm}^{-2}/\text{density}$ ).



Near Cerro Negro, 1992

Photo by C. Connor



Cerro Negro, 1968. Photo by William Melson, 1968 (Smithsonian Institution). Obtained from GVP Website.

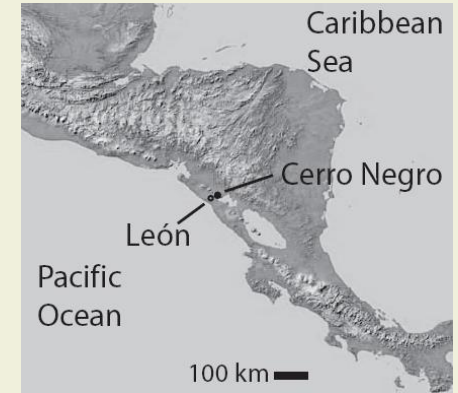


Anchorage, Alaska during an eruption of Mount Spur, 130 kilometers (80 miles) away. Photo by Richard Emanuel, USGS.

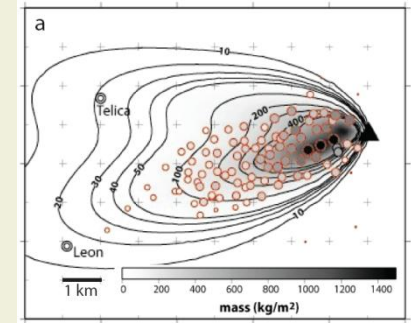
## Example Eruption: Cerro Negro, 1992

**Cerro Negro is a small basaltic cinder cone within the Central American volcanic arc that formed in 1850 and has erupted approximately 24 times since. Eruptions typically last hours to days and are characterized by columns extending 4 – 8 km into the atmosphere. Tephra falls from numerous Cerro Negro eruptions have impacted local residents and the population center in León, Nicaragua, ~30 km west-southwest of the vent. (Steady trade winds tend to advect tephra towards the WSW).**

**Cerro Negro erupted in 1992 after 21 years of quiescence. The 7 km eruption column was observed by the Instituto Nicaragüense de Estudio Territoriales (INETER). The activity was accompanied by dramatic widening of the vent and erosion of the cinder cone. The 1992 eruption consisted of two phases. The initial phase, lasting approximately 7 hours, was characterized by an energetic plume reaching approximately 7 km into the atmosphere. The second phase, lasting approximately 17 hours, was characterized by a weak, bent over plume of column height 1 – 4 km [Connor et al., 1993].**



**Location Map**



**Isopach map created by inverting trench data with Tephra2 (inversion edition)**



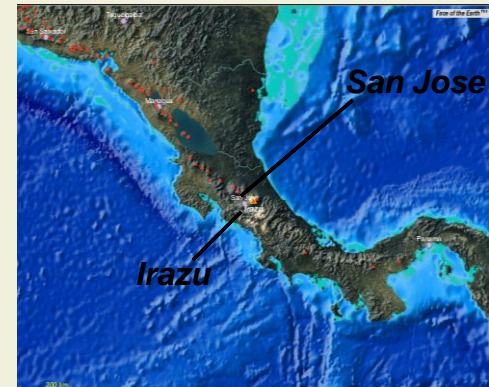


## Example Eruption: Irazu, 1963

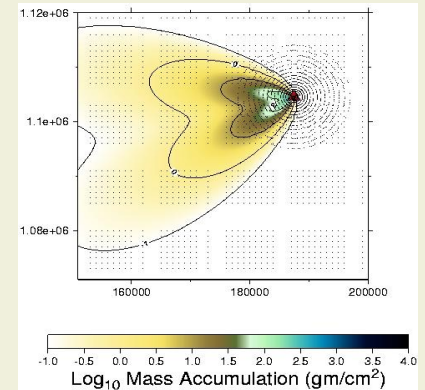
*Irazu Volcano, located in Cost Rica, is recognized as one of the largest and most active volcanoes in South America. The volcano has several active craters, the largest of which delves 900 feet (275 m) into the edifice. Eruptions here typically range in explosivity from VEI 1 – 3.*

*Ash fall from the 1963-65 eruption of Irazú affected both populated areas and agricultural lands. In the weeks following the beginning of the eruption on March 13, 1963 dense ash fall at times restricted visibility to 30 m in the capital city of San José, paralyzing traffic. Ash fall caused extensive damage to farmlands, and about 2000 dairy cows either died of disease and malnutrition or had to be killed. Remobilization of the heavy ash blanket during the rainy season caused mudflows that destroyed houses, roads, and bridges, and caused fatalities. [information taken from GVP Website].*

*GUI data taken from*



**Location Map**



**Isopach map created by gridding Tephra2 output**

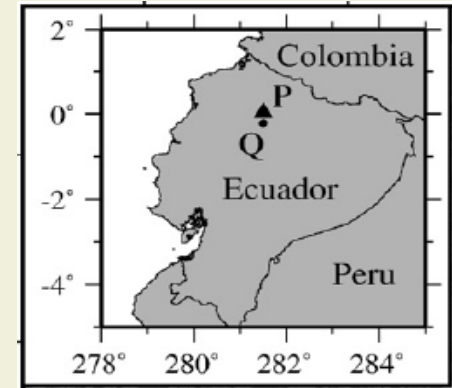


## Example Eruption: Pululagua, 2350BP

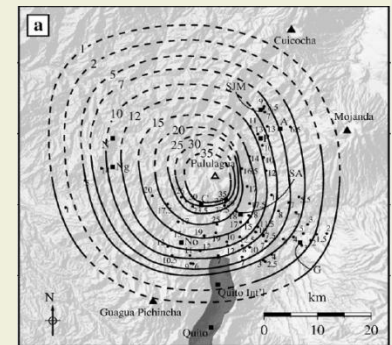
*Pululagua is a relatively low, forested volcano immediately north of the equator, about 15 km north of Quito. The volcano contains a 5-km-wide summit caldera narrowly breached to the west and partially filled by a group of dacitic lava domes. Older pre-caldera lava domes are found on the eastern, SE, and southern sides of the caldera, with a group of younger pre-caldera lava domes primarily on the eastern side. Four post-caldera domes rises up to 480 m above the caldera floor. Large explosive eruptions producing pyroclastic flows took place during the late Pleistocene and Holocene. Caldera formation took place during a series of eruptions lasting 150-200 years beginning about 2650 radiocarbon years ago. The latest dated eruption occurred from post-caldera lava domes about 1670 years ago and produced lava flows and pyroclastic flows. The GUI example models one fallout later, data from Volenkik, 2010.*

*Information from GVP Website:*

*<http://www.volcano.si.edu/world/volcano.cfm?vnum=1502-011>*



*Location Map*



*Isopach map created by inverting trench data with Tephra2 (inversion edition)*



## A bit more Math: Advection-Diffusion

The numerical simulation of tephra accumulation can be expressed by a simplified mass-conservation equation of the type [Suzuki., 1983]:

$$\frac{\partial C_j}{\partial t} + w_x \frac{\partial C_j}{\partial x} + w_y \frac{\partial C_j}{\partial y} - v_{i,j} \frac{\partial C_j}{\partial z} = K \frac{\partial^2 C_j}{\partial x^2} + K \frac{\partial^2 C_j}{\partial y^2} + \Phi \quad \text{Equation 1}$$

where,  $x$  is positive in the mean downwind direction,  $y$  is the mean cross-wind direction, and  $z$  is vertical;  $C_j$  is the mass concentration of particles ( $\text{kg m}^{-3}$ ) of a given particle size class,  $j$ ;  $w_x$  and  $w_y$  are the  $x$  and  $y$  components of the wind velocity ( $\text{m s}^{-1}$ ) and vertical wind velocity is assumed to be negligible;  $K$  is a horizontal diffusion coefficient for tephra in the atmosphere ( $\text{m}^2 \text{s}^{-1}$ ) and is assumed to be constant and isotropic ( $K = K_x = K_y$ );  $v_{i,j}$  is the terminal settling velocity ( $\text{m s}^{-1}$ ) for particles of size class,  $j$ , as these particles fall through a level in the atmosphere,  $i$ ; and  $\Phi$  is the change in particle concentration at the source with time,  $t$  ( $\text{kg m}^{-3} \text{s}^{-1}$ ). The terminal settling velocity,  $v$ , is calculated for each particle size,  $j$ , released from a height level,  $i$ , as a function of the particle's Reynolds number, which varies with atmospheric density [Bonadonna et al., 1998]. Horizontal wind velocity is allowed to vary as a function of height in the atmosphere, but is assumed to be constant within a specific atmospheric level.

An analytical solution to the mass conservation equation can be written as:

$$f_{i,j}(x, y) = \frac{1}{2\pi\sigma_{i,j}^2} \exp\left(-\frac{(x - \bar{x}_{i,j})^2 + (y - \bar{y}_{i,j})^2}{2\sigma_{i,j}^2}\right) \quad \text{Equation 2}$$

for a line plume source where  $\bar{x}_{i,j}$  and  $\bar{y}_{i,j}$  are the coordinates of the center of the bivariate Gaussian distribution:

$$\bar{x}_{i,j} = x_i + \sum_{k=0}^{z_i} \frac{w_{x,k} \Delta z_k}{v_{j,k}} \quad \bar{y}_{i,j} = y_i + \sum_{k=0}^{z_i} \frac{w_{y,k} \Delta z_k}{v_{j,k}}$$

In this model the atmosphere is layered such that  $w_{x,k}$  and  $w_{y,k}$  are true mean downwind and crosswind components of wind velocity in layer  $k$ ;  $\Delta z_k$  is the thickness of layer  $k$ , and  $v_{j,k}$  is the settling velocity for the particle size fraction  $j$  in layer  $k$ . This settling velocity depends on particle density, shape, and physical properties of the atmosphere.

## A bit more Math: Advection-Diffusion

The parameter  $\sigma_{i,j}^2$  is the variance of the Gaussian distribution, which is controlled by atmospheric diffusion and horizontal spreading of the plume [Suzuki, 1983]. Effectively, the use of  $\sigma_{i,j}^2$  in Equation 2 lumps complex plume and atmospheric processes into a single parameter. This greatly simplifies the model, making it much easier to implement, but also ignores processes that can affect tephra dispersion such as the structure of the volcanic plume and its interaction with the atmosphere.

The source term of these models is an estimate of the mass per unit time released from the eruptive column at a given height. Once particles leave the column, the type of diffusion they experience is dependent on their size. For relatively coarse particles with relatively short particle fall-times ( $t_{i,j}$ ) diffusion is linear (Fick's law) and the variance is described by [Suzuki, 1983]:

$$\sigma_{i,j}^2 = 2K(t_{i,j} + t'_j) \quad \text{Equation 3}$$

Where  $t'_j$  is the horizontal diffusion time in the vertical plume. This diffusion model strongly depends on the choice of the diffusion coefficient,  $K$ , for large particles.

For fine particles with long settling times, a power-law diffusion model is used [Bonadonna, 1998]. Diffusion for these particles strongly depends on the particle fall time and the horizontal diffusion time of the ascending plume [Suzuki, 1983]. These particles settle far from the volcano.

Ultimately, particles leaving the plume sink through the atmosphere to the ground. Dispersal patterns generated by advection-diffusion models are especially sensitive to total mass of erupted material and, for proximal deposits, column height [Scollo et al., 2009]. Wind direction and velocity also have a significant effect on deposits.

## **Additional Reading:**

**Suzuki's (1983) model computes the mass of tephra deposited at a location relative to the eruption source using an analytic solution to the diffusion - advection equation and a line source for tephra in the eruption column. A complete description of the original mathematical development is available in:**

**Suzuki, T., 1983. A theoretical model for dispersion of tephra, in: D. Shimozuru and I. Yokoyama (eds) *Arc Volcanism: Physics and Tectonics*, Terra Scientific Publishing, Tokyo, 95-116.**

**The model used here is slightly modified. See:**

**Connor, C.B., B.E. Hill, B. Winfrey, N.M. Franklin, and P.C. LaFemina, 2001, Estimation of volcanic hazards from tephra fallout, *Natural Hazards Review*, 2: 33-42.**

**Bonadonna, C., C.B. Connor, B.F. Houghton, L. Connor, M. Byrne, A. Laing, and T. Hincks, 2005. Probabilistic modeling of tephra dispersion: hazard assessment of a multi-phase eruption at Tarawera, New Zealand, *Journal of Geophysical Research*, 110 (B03203).**

**Further Reading:**

**Barberi, F., G. Macedonio, M.T. Pareschi, and R. Santacroce, 1990. Mapping the tephra fallout risk: an example from Vesuvius, Italy, *Nature*, 344, 142-144.**

**Costa, A., G. Macedonio and A. Folch, 2006. A three-dimensional Eulerian model for transport and deposition of volcanic ashes, *Earth and Planetary Science Letters*, 241 (3-4), 634-647.**

**D'Amours, R., 1998. Modeling the ETEX plume dispersion with the Canadian emergency response model, *Atmospheric Environment*, 32 (24), 4335-4341.**

**Heffter, J.L., and B.J.B. Stunder, 1993. Volcanic Ash Forecast Transport and Dispersion (Vaftad) Model, *Weather and Forecasting*, 8 (4), 533-541.**

**Hurst, A.W., and R. Turner, 1999. Performance of the program ASHFALL for forecasting ashfall during the 1995 and 1996 eruptions of Ruapehu volcano, *New Zealand Journal of Geology and Geophysics*, 42 (4), 615-622.**

**Macedonio, G., M.T. Pareschi, and R. Santacroce, 1998. A numerical simulation of the Plinian fall phase of 79 AD eruption of Vesuvius, *Journal of Geophysical Research-Solid Earth and Planets*, 93 (B12), 14817-14827.**

**Volentik A.C.M., Bonadonna C., Connor C.B., Connor L.J., and Rosi M. Modeling tephra dispersal in absence of wind: insights from the climactic phase of the 2450 BP Plinian eruption of Pululagua volcano (Ecuador). *Journal of Volcanology and Geothermal Research*, 193(1-2), pp. 117-136, doi:10.1016/j.jvolgeores.2010.03.011**