Strain rates to stresses

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Based on Cuffey & Paterson Chapter 3

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Start with the expression for effective strain rate, $\dot{\epsilon}_E$, which is Cuffey & Paterson Eq. 3.17 on page 59:

$$\dot{\epsilon}_E = \sqrt{\frac{1}{2}\left(\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2 + \dot{\epsilon}_{zz}^2\right) + \dot{\epsilon}_{xz}^2 + \dot{\epsilon}_{xy}^2 + \dot{\epsilon}_{yz}^2} \tag{1}$$

I will ignore the vertical components (assume near 0) and replace the normal strain rates with the principal strain rates, $\dot{\epsilon}_1$ and $\dot{\epsilon}_3$, which is what I have maps of:

$$\dot{\epsilon}_E = \sqrt{\frac{1}{2}\left(\dot{\epsilon}_1^2 + \dot{\epsilon}_3^2\right) + \dot{\epsilon}_{xy}^2} \tag{2}$$

I will do some sensitivity testing on the shear strain rate $\dot{\epsilon}_{xy}$. My base plan would be to set $\dot{\epsilon}_{xy} = 0$.

Now I am ready to get the stresses τ_{jk}, which is Cuffey & Paterson Eq. 3.25 on page 61:

$$\tau_{jk} = \frac{1}{A^{1/n}} (\dot{\epsilon}_E)^{(1-n)/n}\ \dot{\epsilon}_{jk} \tag{3}$$

Here, A is the flow law parameter, generally 3.5×10^{-25} Pa^{-3}s^{-1} (Cuffey & Paterson Table 3.4, p. 75), and the flow law exponent $n = 3$. (Note to self: the strain rates I have are in yr^{-1}, not s^{-1}, so I will need to convert A accordingly.)

I will aim for the principal stresses σ_1 and σ_3, which are respectively compressional (-) and extensional (+). They are full stresses (σ) rather than the general or shear stresses (τ).

$$\sigma_1 = \frac{1}{A^{1/n}} (\dot{\epsilon}_E)^{(1-n)/n}\ \dot{\epsilon}_1 \tag{4}$$

$$\sigma_3 = \frac{1}{A^{1/n}} (\dot{\epsilon}_E)^{(1-n)/n}\ \dot{\epsilon}_3 \tag{5}$$