# Probabilistic Hazard Map Emulator Overview

UB

February 15, 2016

# Outline

- Probabilistic Hazard Map Emulator Construction Overview
- Probabilistic Hazard Map Emulator Construction Workflow

# Hazard Map Emulation Motivation and Challenges

- Analyze hazards over a terrain over a period time following a premonitory event
- Goal for this current study is to produce a hazard map that displays the probability of a flow depth reaching critical height
- Currently there are not many procedures in place for obtaining field data from a premonitory event within hours of the event
- Goal is to support on-line access and dissemination of the the probability of hazard map workflow through the VHub platform

# Computer Models

- Need a computer implementation of mathematical model
- Direct implementation of computer model is very costly
- Need adaptively designed approximations for the computer model
- Need to handle uncertainty in the input parameters to the computer model

# Simulator Parameter Uncertainty

- In real world problems there is some degree of uncertainty
- Simulators: Deterministic computer models
- Emulators: Statistical models of simulators
- Focus on parameter uncertainty: The uncertainty of the simulator output due to uncertainty in its inputs
- One way to to quantify the uncertainty is to use sampling which requires multiple runs of the simulator
- Each run of TITAN2D takes 20 minutes on a single processor so Monte Carlo type sampling is considered too expensive

# TITAN2D Simulator

- TITAN2D Simulator provides the deterministic computer model
- Geophysical flow computer model developed by the Geophysical Mass Flow group at UB
- Simulates granular flows such as debris avalanches and landslides over digital elevation models (DEMs) which are 3D representation of a terrain's surface
- TITAN2D combines numerical simulations of a flow with digital elevation data of natural terrain supported through a Geographical Information System (GIS) interface such as GRASS
- Employs adaptive mesh refinement

# TITAN2D Governing "Shallow Water" Equations

- Hyperbolic system of equations solved using a parallel adaptive mesh
- $U_t + F(U)_x + G(U)_y = S(U)$
  - $U$ is a vector of conserved state variables
  - $F$ is a vector of mass and momentum fluxes in the x-direction
  - $G$ is a vector of mass and momentum fluxes in the y-direction
  - $S$ is a vector of driving and dissipative force terms
- System is solved numerically for flow depth and a depth-averaged velocity at every grid point and time step

# Bayes Linear Emulator

- The Bayes Linear Emulator is a very important step in the construction of the probabilistic hazard map emulator
- The emulator attempts to fit a piecewise polynomial through the already available data
- The emulator estimates the means and variances of the inputs for the uncertain parameters using Bayesian Linear Regression

# Bayesian Approach

- The full Bayesian approach requires knowledge about a joint probability distribution
- Estimates some unknown stochastic process represented by a set of data
- Employs Bayes Rule
  - $P(X|Y) = \dfrac{P(X)P(Y|X)}{P(Y)}$
- In some cases the joint probability distribution is not known or may be arbitrarily specified

# Bayes Linear Method I - Adjusting Beliefs: Concepts and Properties

- Belief specification
- $C = (X_1, X_2, ...)$ Random variables from which statements of uncertainty are made
- For each $X_i, X_j \in C$ specify
  - $E(X_i)$ Belief as to the value of $X_i$
  - $Var(X_i)$ Degree of Confidence in the belief
  - $Cov(X_i, X_j)$ Judgment on the relationship between the quantities

# Bayes Linear Method I - Adjusting Beliefs: Concepts and Properties Con't

- Belief structure

- Construct linear space $\langle C \rangle$ consisting of all linear combinations of random quantities

  - $h_0 X_o + h_1 X_{i_1} + ... + h_k X_{i_k}$

- Belief structure allows restriction by choice of base to any linear subspace of this collection

- Probabilistic hazard map emulator is constructed over $\langle C \rangle$

# Bayes Linear Method I - Adjusting Beliefs: Concepts and Properties Con't

- $D = \{D_1 ... D_k\}$ Observed members of C
- $B = \{B_1 ... B_r\}$ Uncertain members of C
- Adjusted Expectation, Var, and Cov for $X \in B$
  - $E(X|D) = \sum_{i=0}^{k} h_i D_i$
  - That minimizes $E((X - \sum_{i=0}^{k} h_i D_i)^2) \Rightarrow$
  - $E(X|D) = E(X) + Cov(X, D) Var(D)^{-1}(D - E(D))$
  - $VAR(X|D) = E((X - E(X|D)^2)$
  - $COV(X, Y|D) = E((X - E(X|D))(Y - E(Y|D)))$

# Piecewise Linear Ensemble of Emulators

- Multi mini-emulator formulation guarantees a positive definite matrix
- Correlation for using existing methods would have millions of elements in each direction
- Using an ensemble of mini-emulators
- Mini-emulators correlation matrices on the order of hundred per side vice one million per side
- One mini-emulator for each of the samples
- Piecewise linearly interpolate mini-emulators

# Tessellations

- Tessellation of sample points is performed to generate a set of triangles whose nodes are samples, uses Delaunay triangularization
- A mini-emulator centered about each sample is constructed using only those samples in the neighborhood of its central sample
- Mini-emulators adjusted mean and variance are calculated for arbitrary re-sample points
- Adjusted means and variances of the min-emulators combined in a weighted sum where sum of weights is 1
- Aggregate mini-emulators into a maco-emulator and re-sample the macro-emulator to produce the probability of a hazard map (PHM)

# Tessellation Example

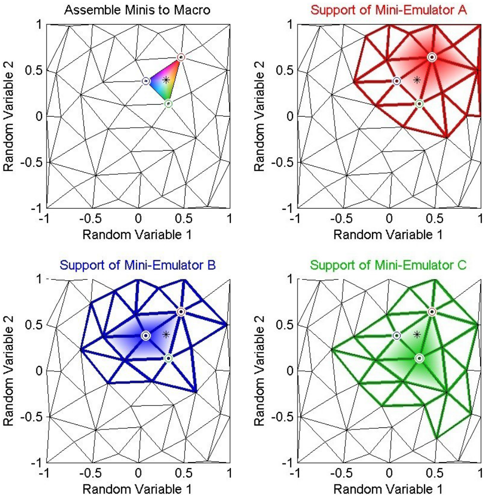
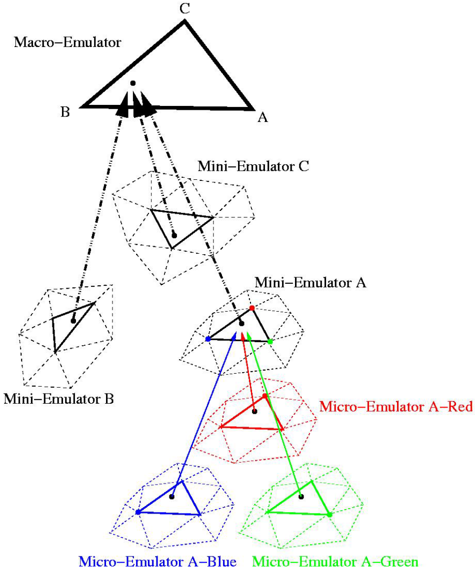



Figure : Example Tessellation

# Ensemble Example



# Probabilistic Hazard Map Emulator Workflow Diagram

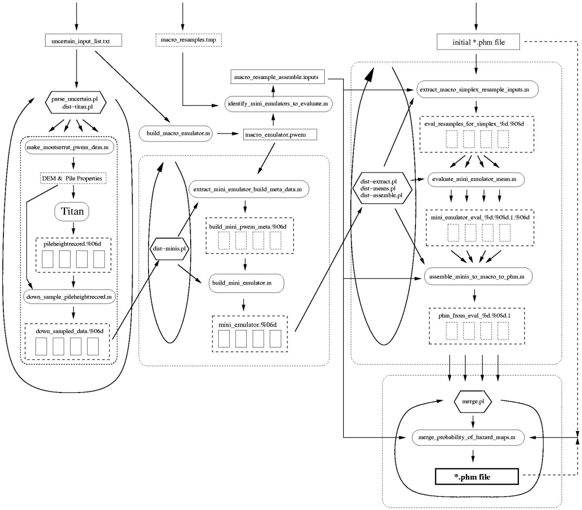
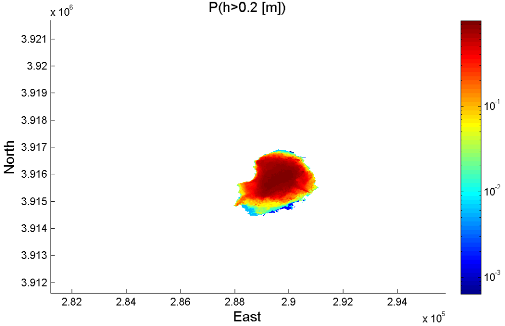


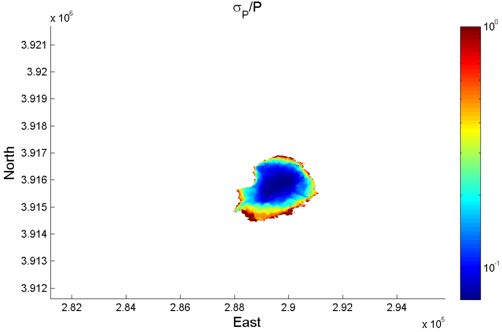

Figure : Probabilistic Hazard Map Emulator Workflow

# Example Probabilistic Hazard Map




- Probability that a flow will exceed 0.2 m in depth as a function of position on Mount Fuji, given uncertainty in the DEM and the volume and bed friction input parameters. Titan simulation run time 60 seconds

# Example Probabilistic Hazard Map




- Standard deviation in the estimate that the flow will exceed 0.2 m in depth

# Pegasus Workflow Management System

- Allows management of multi-step computational tasks
- Supports on-line access and dissemination of probability of hazard map emulations through the VHUB platform
- Interfacing with Pegasus via python script
- Allows automation of the probability of hazard map emulation workflow

# Additional References

Predictive Simulation and Model based Hazard Maps of Geophysical Mass Flow (Dissertation),Keith R. Dalbey

Bayes Linear Method I, Adjusting Beliefs: Concepts and Properties, Micheal Goldstein