SOLVING FOR THE GRAVITY FIELD IN COMSOL

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The gravity change, related to density redistribution, can be calculated by solving the following Poisson's differential equation for the gravitational potential *u*:

$$\nabla^2 u = -4\pi G \Delta\rho(x, y, z) \qquad (1)$$

where *G* denotes the universal gravitational constant and $\Delta\rho(x, y, z)$ is the change in the density distribution. The problem is closed imposing the condition of vanishing gravitational potential at infinity. Generally, the temporal gravity change δg determined by differencing repeated gravity measurements is given by:

$$\delta g(x, y, z) = -\frac{\partial u}{\partial z} + \delta g_0 \qquad (2)$$

where δg_0 represents the "free air" gravity change accompanying the displacements of the observation site.

COMSOL is used to compute the gravity changes due to a sphere at a depth of 3000 m with a radius of 500 m (Fig. 1) and a density contrast $\Delta\rho$ of 200 kg/m^3 by solving the equation in (1).

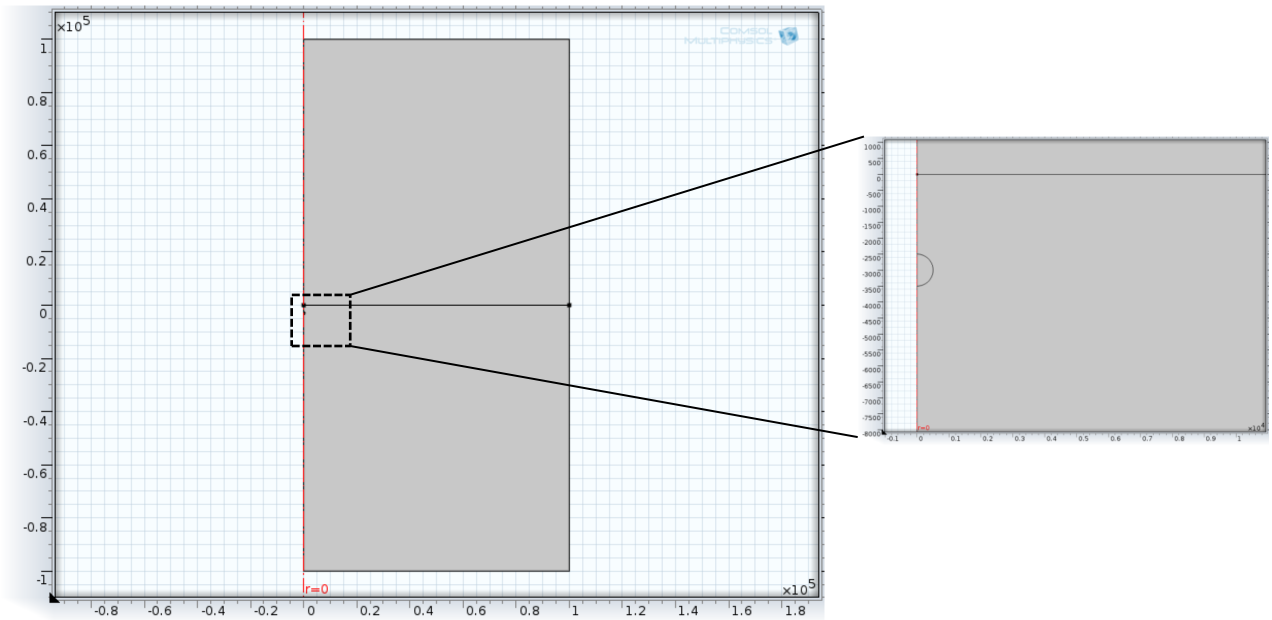

Figure 1 – Computational domain to compute the gravity changes due to a horizontal infinite cylinder (2D problem) or to a sphere (2D axisymmetric problem).

The numerical solutions are compared with the analytical expression:

$$\delta g = G\Delta\rho V \frac{z}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}} \tag{3}$$

COMSOL solution for the 2D axi-symmetry Poisson's equation does not fit the analytical solution. The error arose from the way COMSOL consider the 2D domain. Indeed, the 2D axi-symmetry model is a fully 2D model, which means COMSOL solves:

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} = 4\pi G\Delta\rho \tag{4}$$

Considering the model set-up (Fig. 1), it represents the problem for solving the gravity filed due to an horizontal infinite cylinder (Fig. 2). The equation in a real 2D axi-symmetry problem, analogous to the 3D case, is instead given by:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 4\pi G\Delta\rho \tag{5}$$

For implementing correctly this equation the divergent term 1/r has to be included in the equation as shown in the attached Gravity2D.mph file.

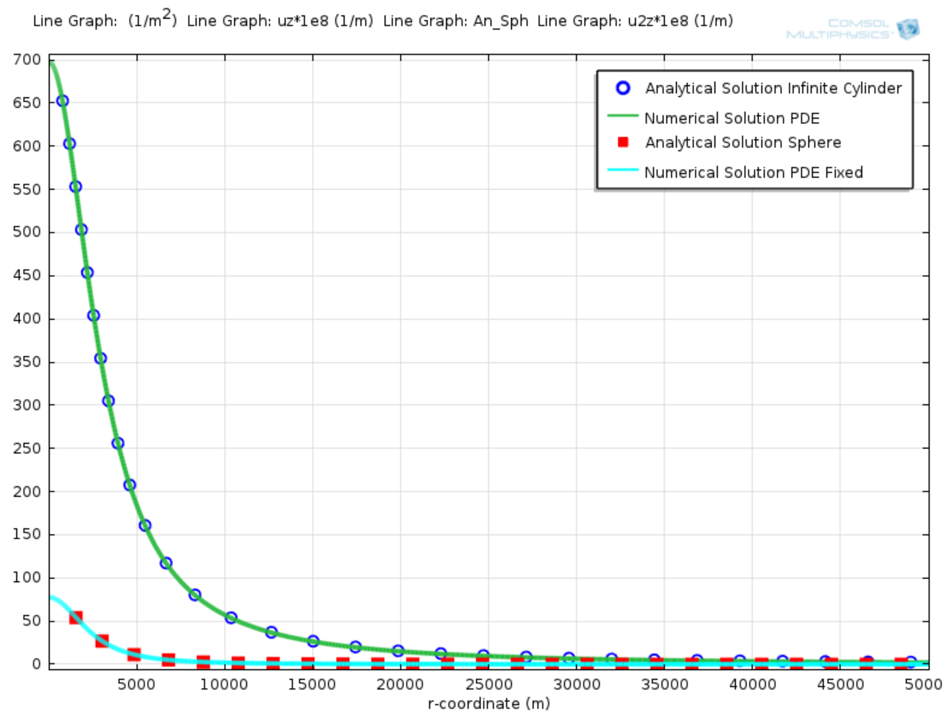

Figure 2 - Analytical solutions of the gravity field due to a sphere (axi-symmetry problem) and to an infinite cylinder (2D problem) are compared with the numerical solutions of COMSOL.