USGS

Part III:
InSAR Deformation Modeling

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Acknowledgement

- Contribution by many colleagues.
- Funding from USGS and NASA.
- Original SAR data are copyrighted ESA, CSA, JAXA , or DLR

Volcano Deformation

1. Many volcanic eruptions are preceded by pronounced ground deformation in response to increasing pressure from magma chambers or to the upward intrusion of magma.

2. Surface deformation patterns can provide important insights into the structure, plumbing, and state of restless volcanoes.

3. Surface deformation might be the first sign of increasing levels of volcanic activity, preceding swarms of earthquakes or other precursors that signal impending intrusions or eruptions.

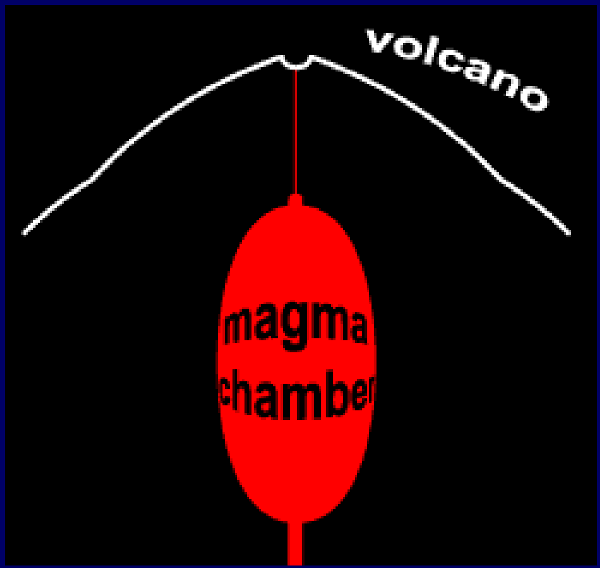
4. Surface deformation provides a critical element on understanding how a volcano work.

Deformation Source

deformation:
what we see (InSAR)

?

magma dynamics:
what we want to know



Magma intrusion

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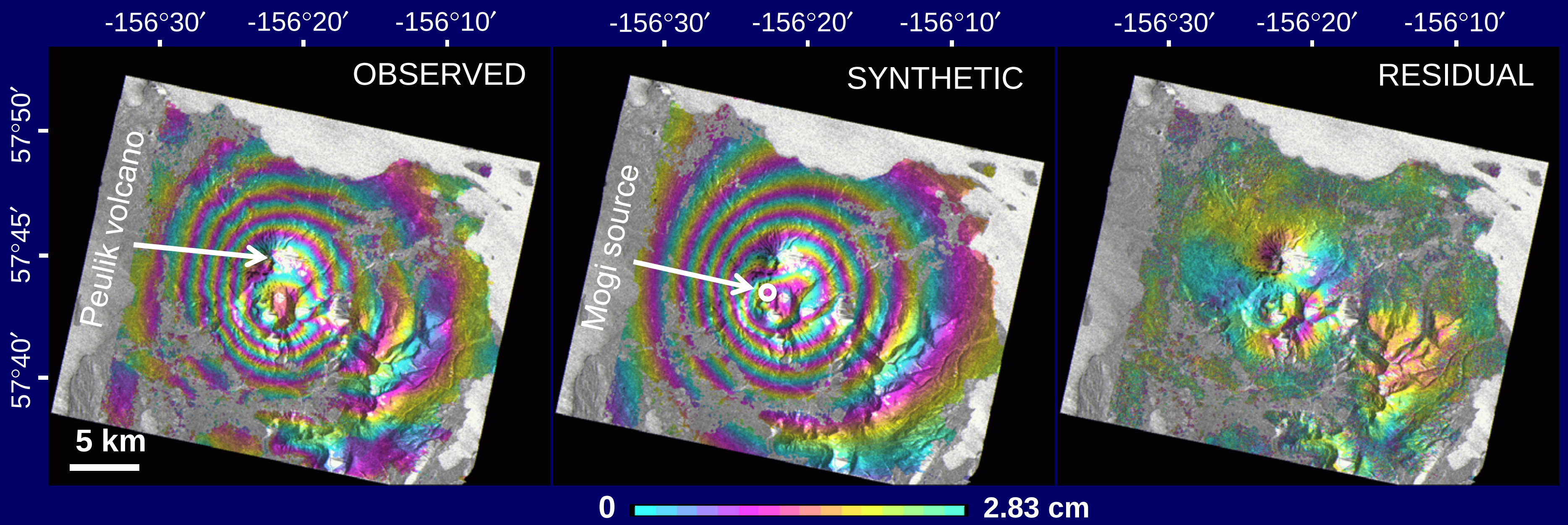
Deformation modeling



Spherical point source (Mogi source)

$$u_i\,(x_1 - x_1', x_2 - x_2', 0) = \frac{(1-\nu)}{\pi}\frac{x_i - x_i'}{R^3}\Delta V$$

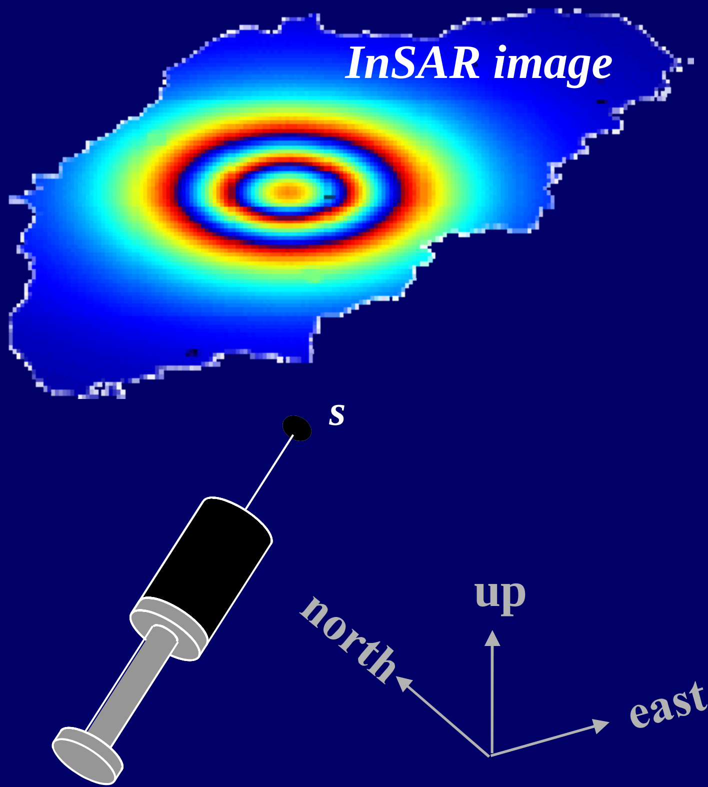
where x'_1, x'_2 , and x'_3 are horizontal locations and depth of the center of the sphere, R is the distance between the sphere and the location of observation (x_1, x_2 , and 0), and ν is the Poisson's ratio of host rock.

Best-fit source parameters:

- The model source is located at a depth of 6.5 ± 0.2 km.
- The calculated volume change of magma reservoir is 0.043 ± 0.002 km^3.

Deformation Modeling

Estimate source characteristics from InSAR deformation data



forward model

design matrix

$$\mathbf{G}\, s = \mathbf{d}$$

source parameters — displacement (vector)

inverse model

$$s = \mathbf{G}^{inv}\, \mathbf{d}$$

Linear Inversion

$$Ax = b$$

design matrix (A) — *model parameters* (x) — *observations* (b)

If the covariance matrix for errors in the observation (*b*) is $\sum_b$, then the weighted least-squares (maximum likelihood) solution for *x* is

$$\hat{x} = [G^T \textstyle\sum_b^{-1} G]^{-1} [G^T \textstyle\sum_b^{-1} b]$$

The covariance matrix for the estimated vector components is

$$\textstyle\sum_x = [G^T \textstyle\sum_b^{-1} G]^{-1}$$

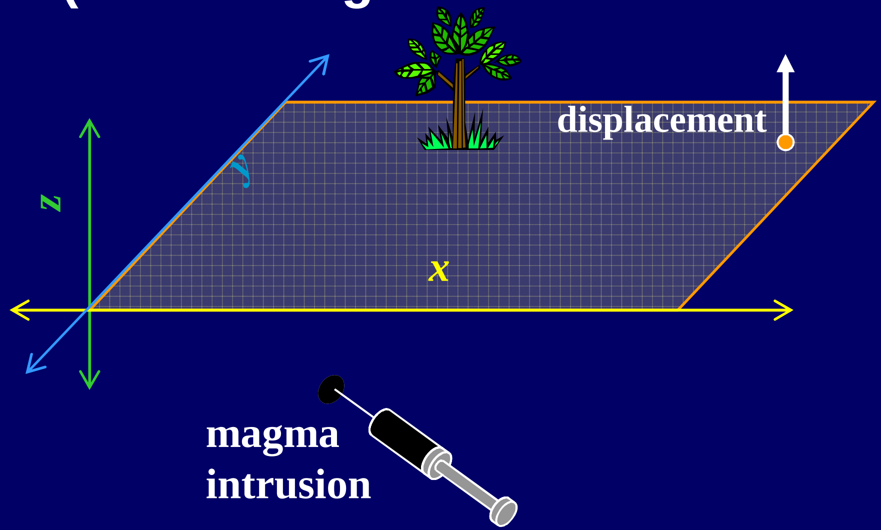
In the case where we assume that observation errors are independent and have equal standard deviations, σ , we get

$$\textstyle\sum_x = \sigma^2 [G^T G]^{-1}$$

The square root of the diagonal terms give the standard errors in parameter estimates

Forward model

Predicts deformation ($\underline{u}$) caused by magma intrusion (relates magma intrusion to deformation)



elasto-static behavior

$$\mu \nabla^2 u_i + \frac{\mu}{(1-2\upsilon)} \left[\frac{\partial^2 u_k}{\partial x_i \partial y_k} \right] = -F_i$$

$$\underline{u} = f(\textit{model parameters})$$

Forward model: point source

A component of deformation vector (u_i) and the displacement at the free surface (x_3=0) takes the form

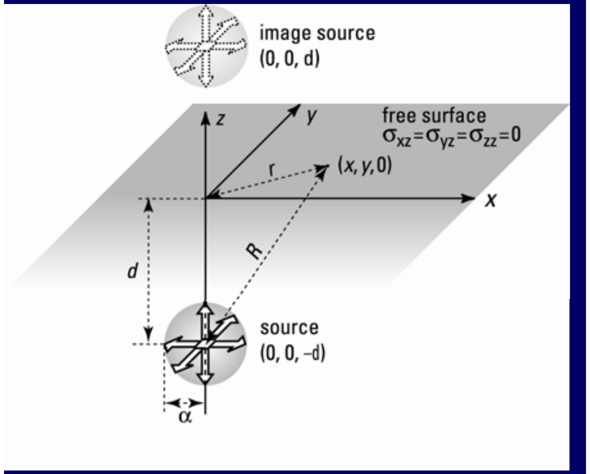
$$u_i(x_1 - x_1', x_2 - x_2', -x_3) = C\frac{x_i - x_i'}{|R^3|}$$

x_i' is a source location, C is a combination of material properties and source strength, and R is the distance from the source to the surface location

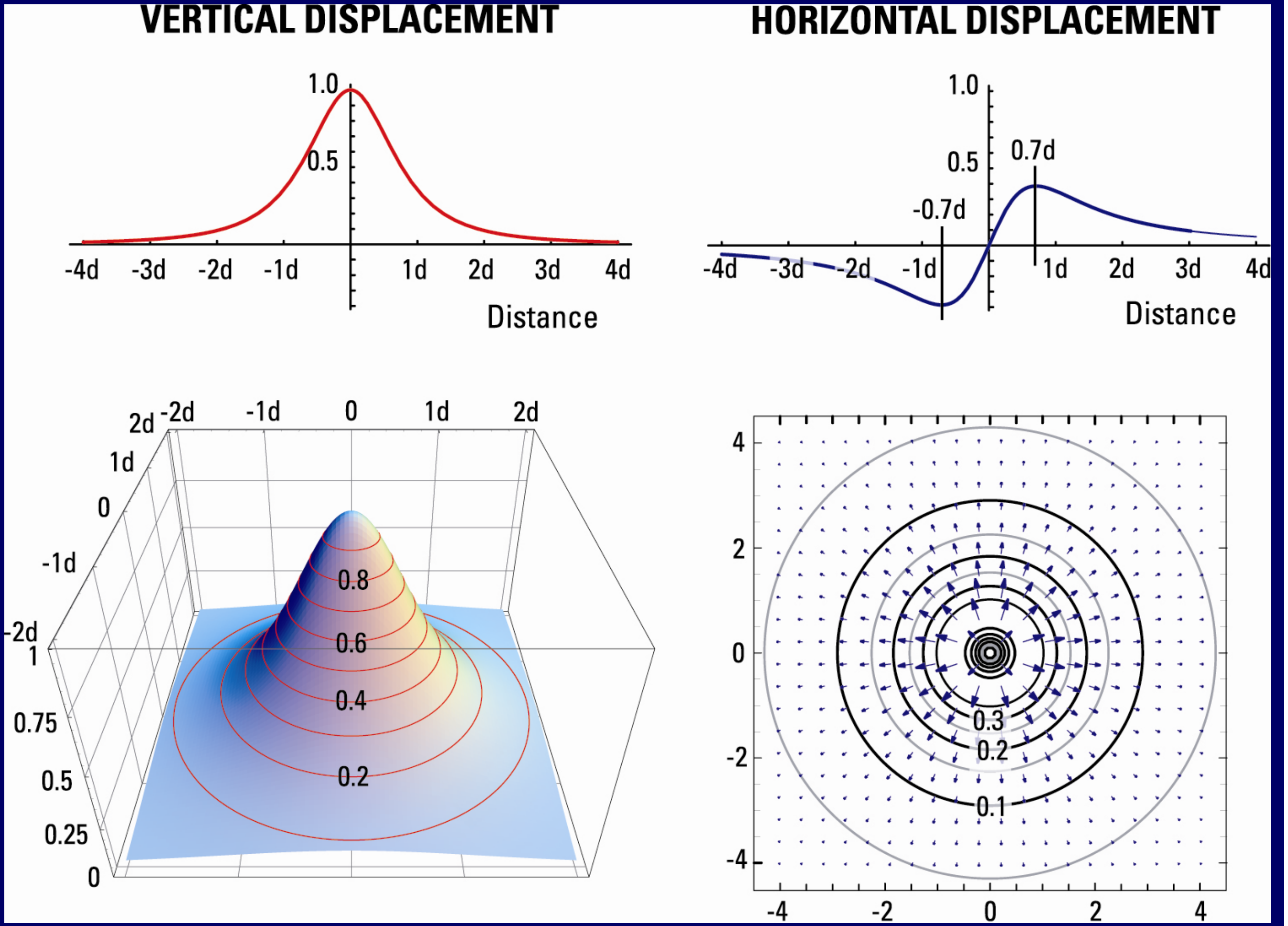
$$C = \Delta P(1-\nu)\frac{r_s^3}{G} = \Delta V\frac{(1-\nu)}{\pi}$$

Δp - change in pressure of magma chamber
ΔV - change in volume of magma chamber
ν - Poisson's ratio
r_s - radius of the sphere
G - shear modulus of country rock

Forward model: point source



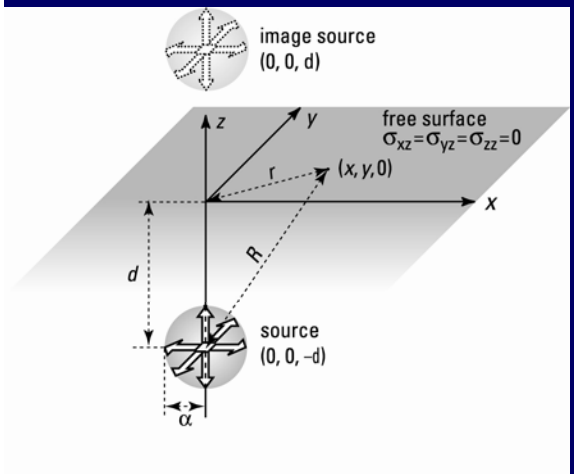
α << d

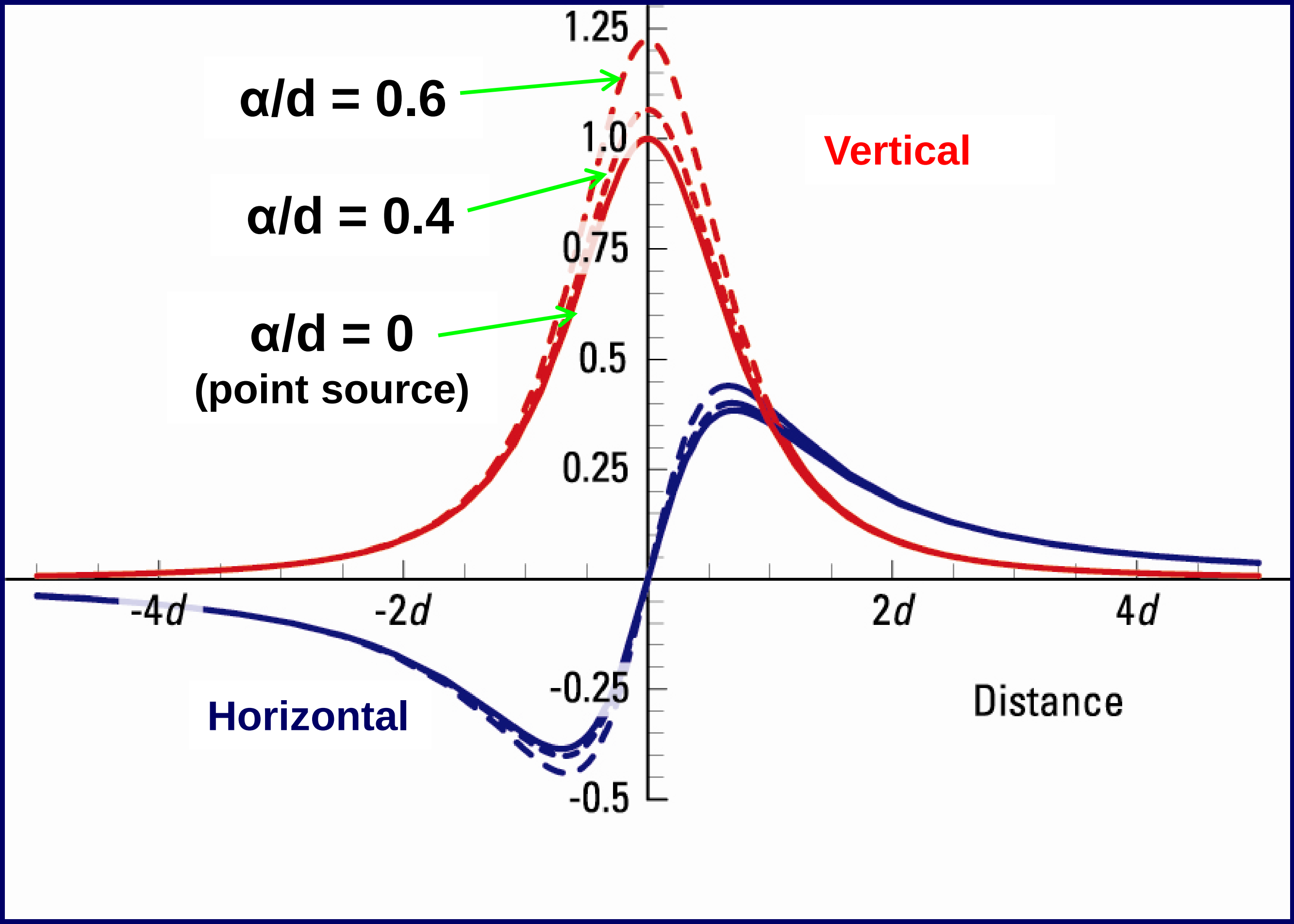


D. Dzurisin, 2007

Courtesy of M. Lisowski

Forward model: spherical source

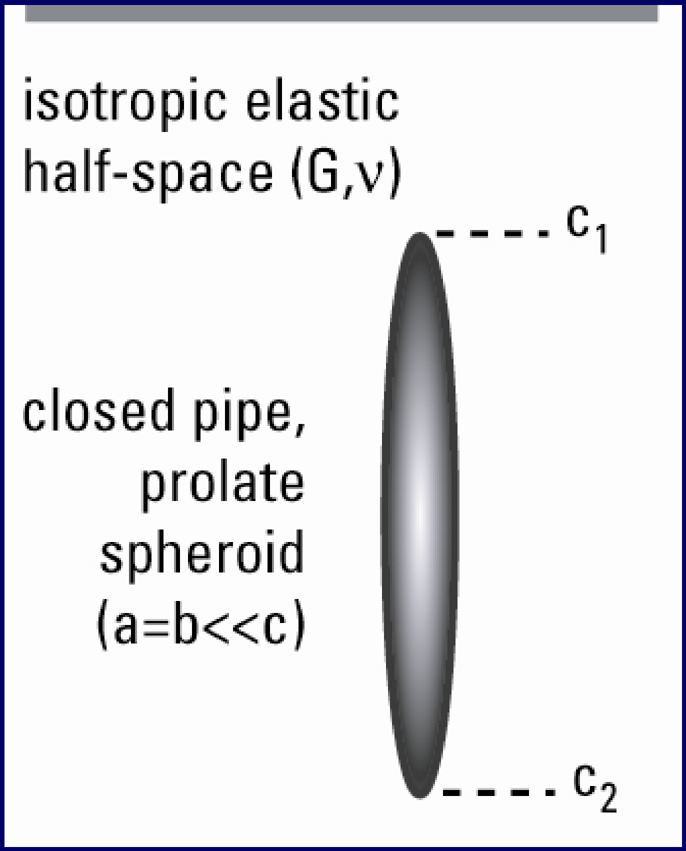


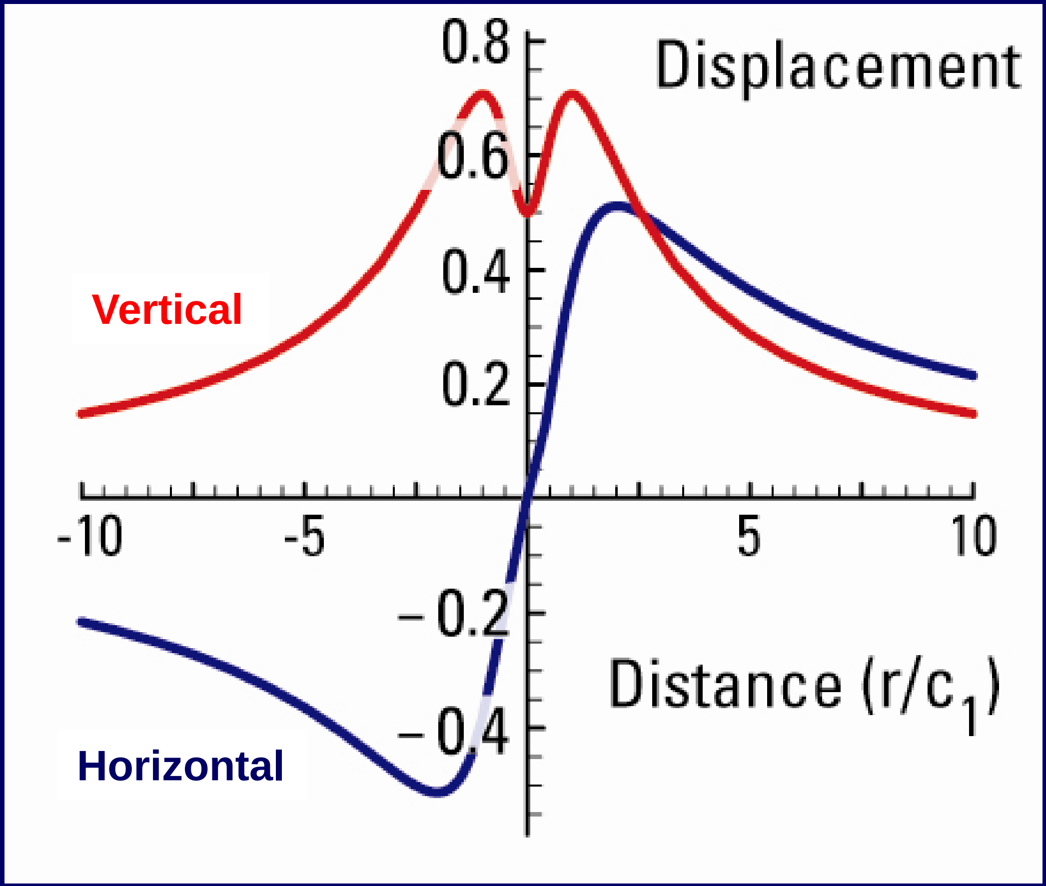


D. Dzurisin, 2007

Courtesy of M. Lisowski

Forward model: closed pipe





C_2 -> infinity

D. Dzurisin, 2007
Courtesy of M. Lisowski

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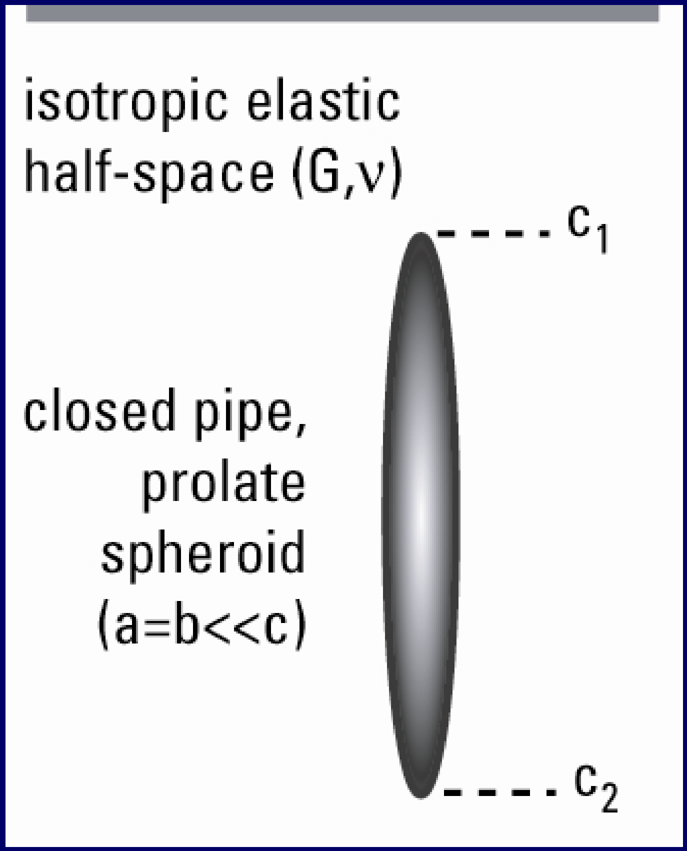
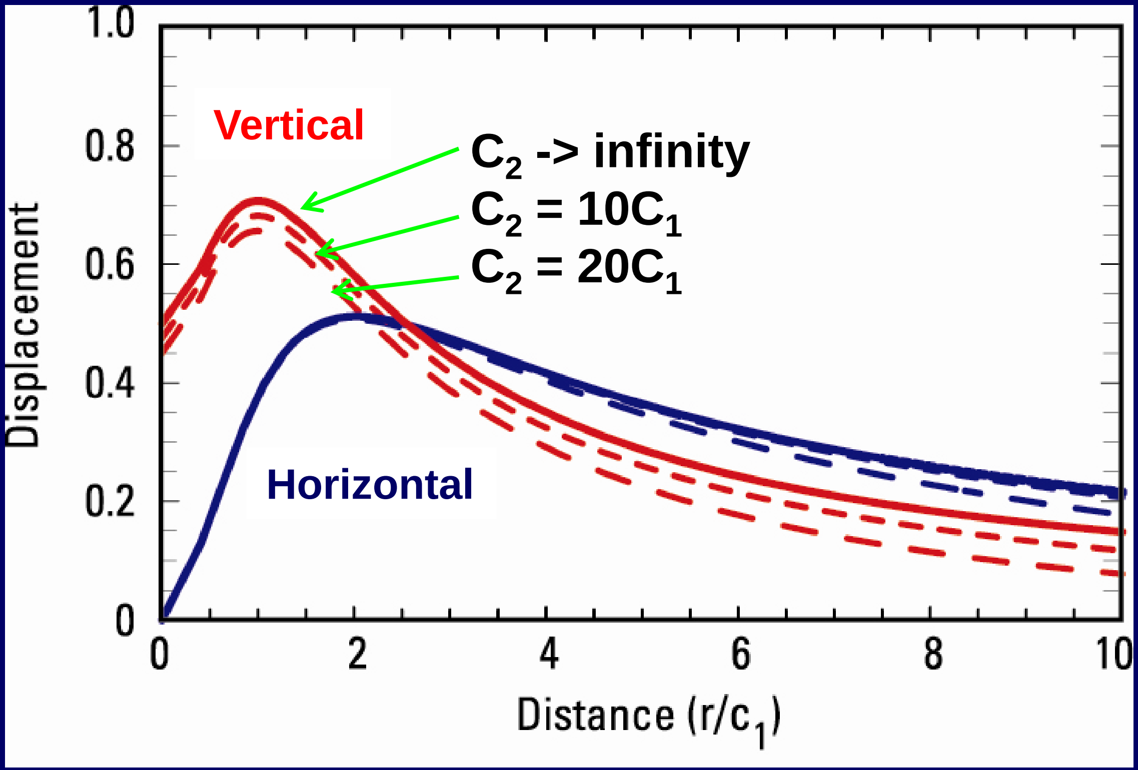
Forward model: closed pipe





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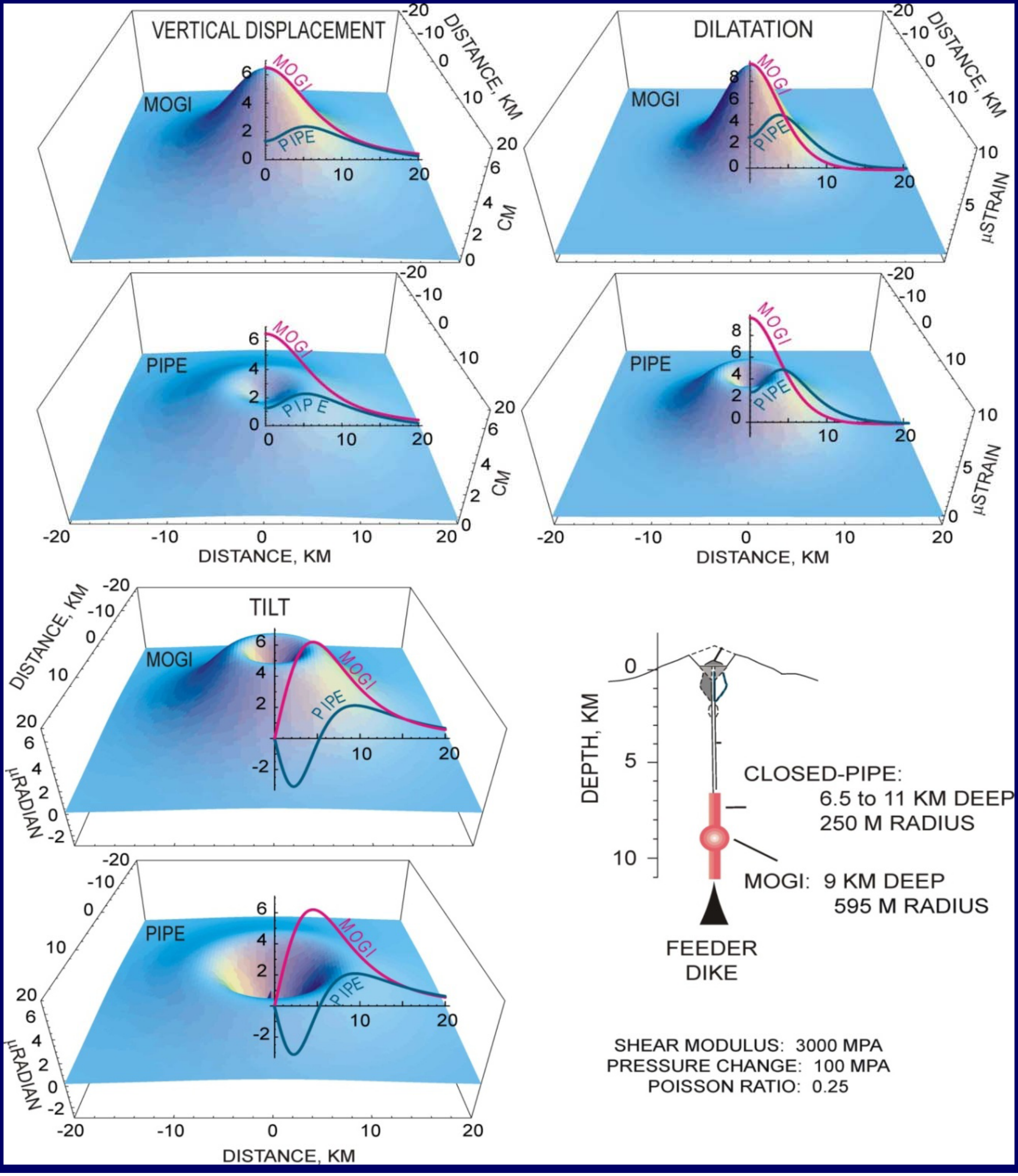
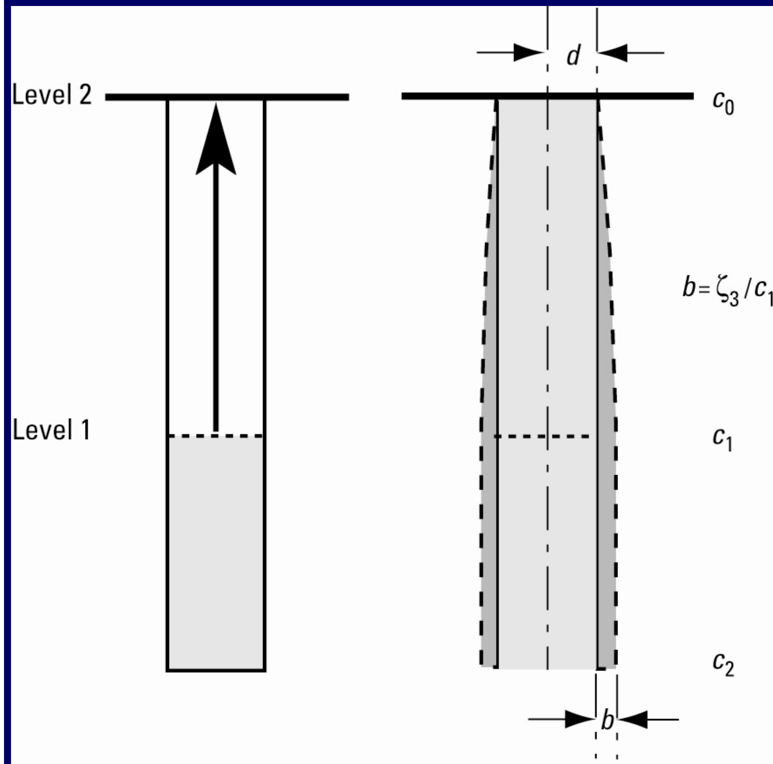
Forward model: pipe vs Mogi

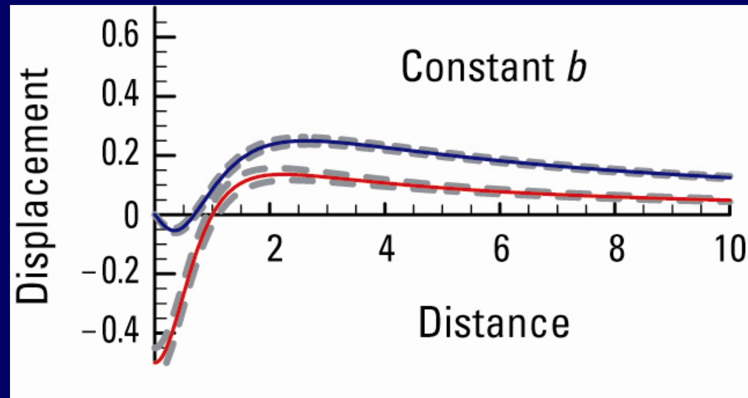


D. Dzurisin, 2007

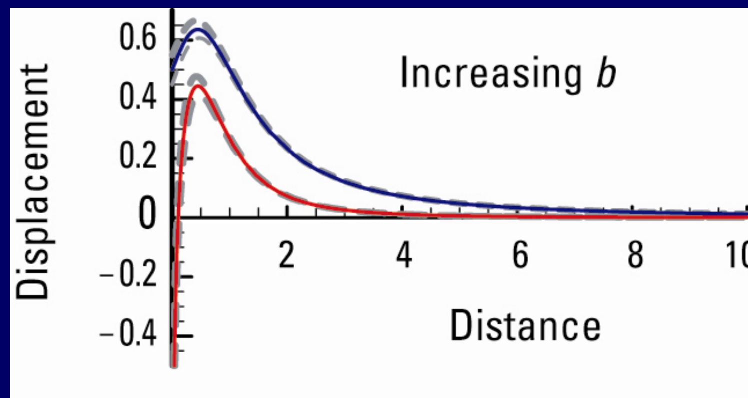
Forward model: open pipe



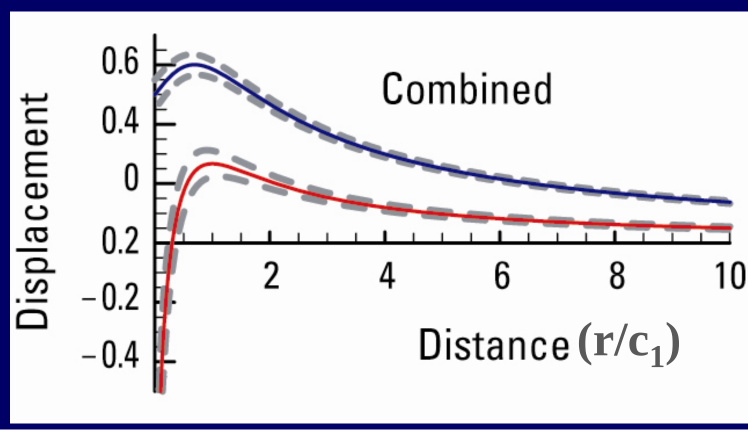
$$b = \frac{\Delta P}{G}\alpha$$



constant pressure change in the lower section of conduit



filling the top portion of the conduit from c1 to surface



combined effect of filling a conduit from c1 to surface

Courtesy of M. Lisowski

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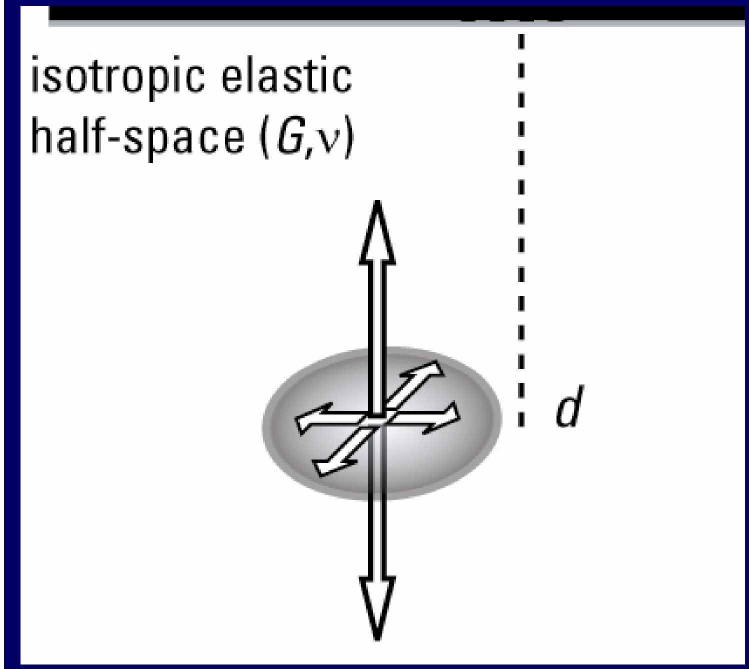
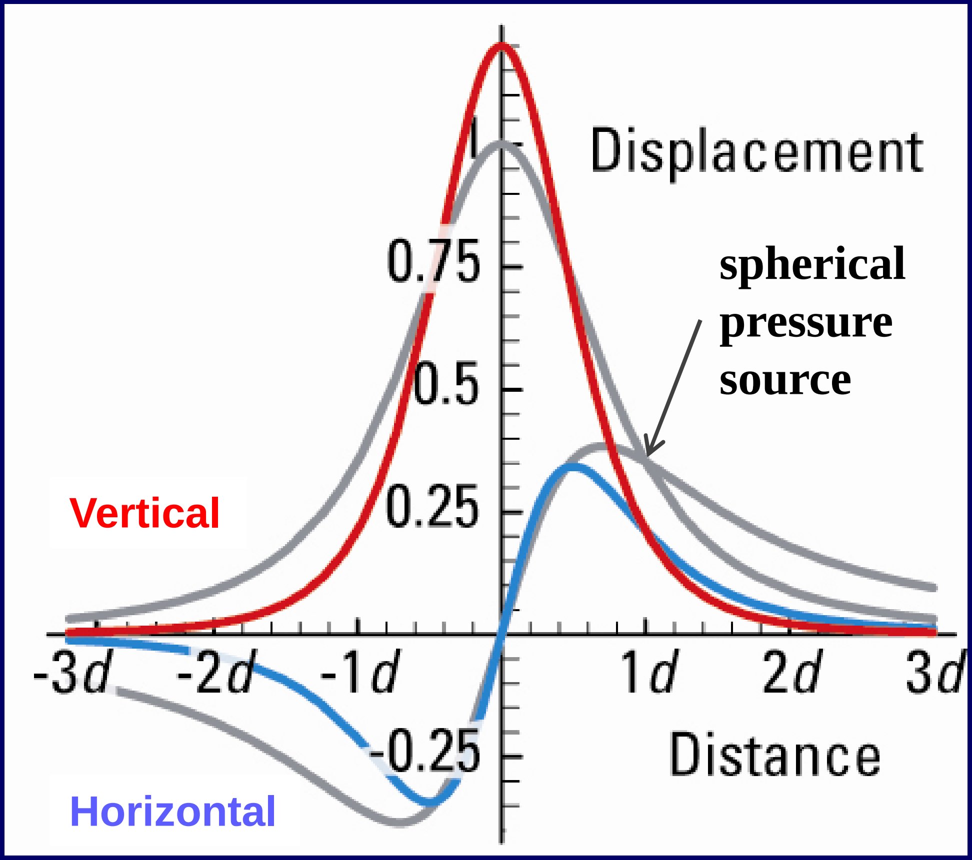
Forward model: sill





Courtesy of M. Lisowski

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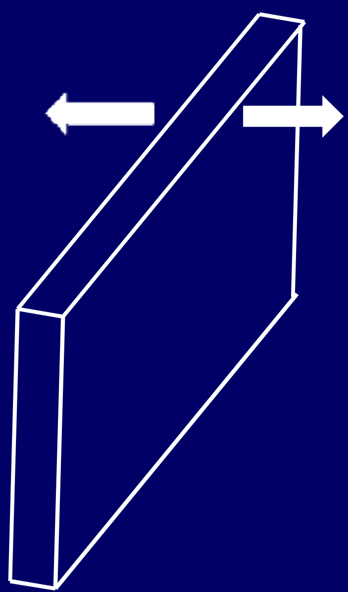
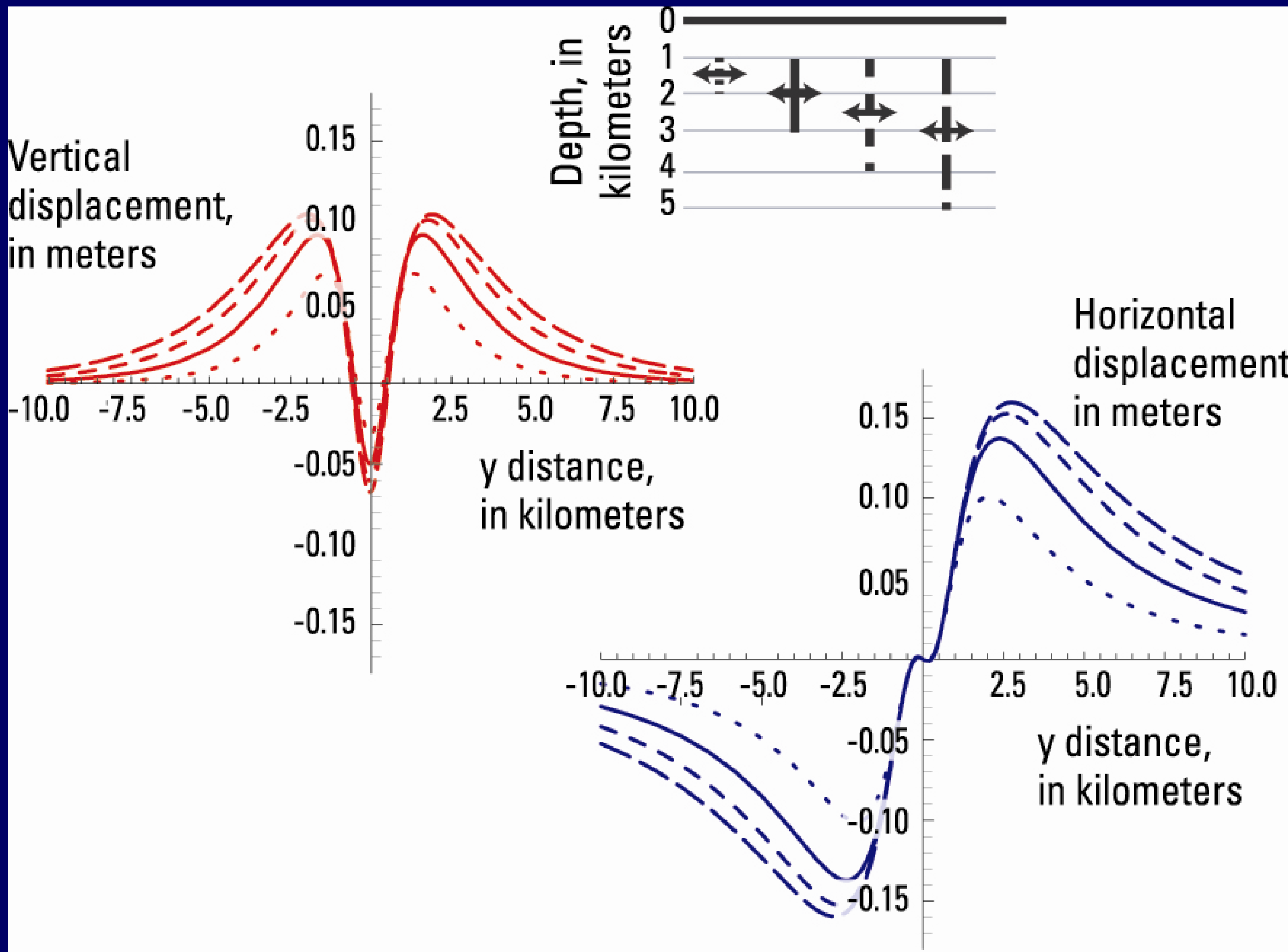
Forward model: dike



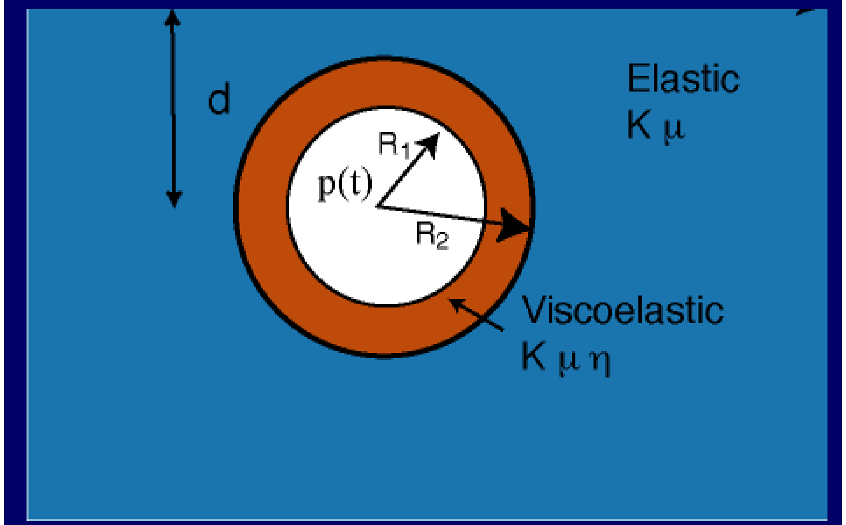


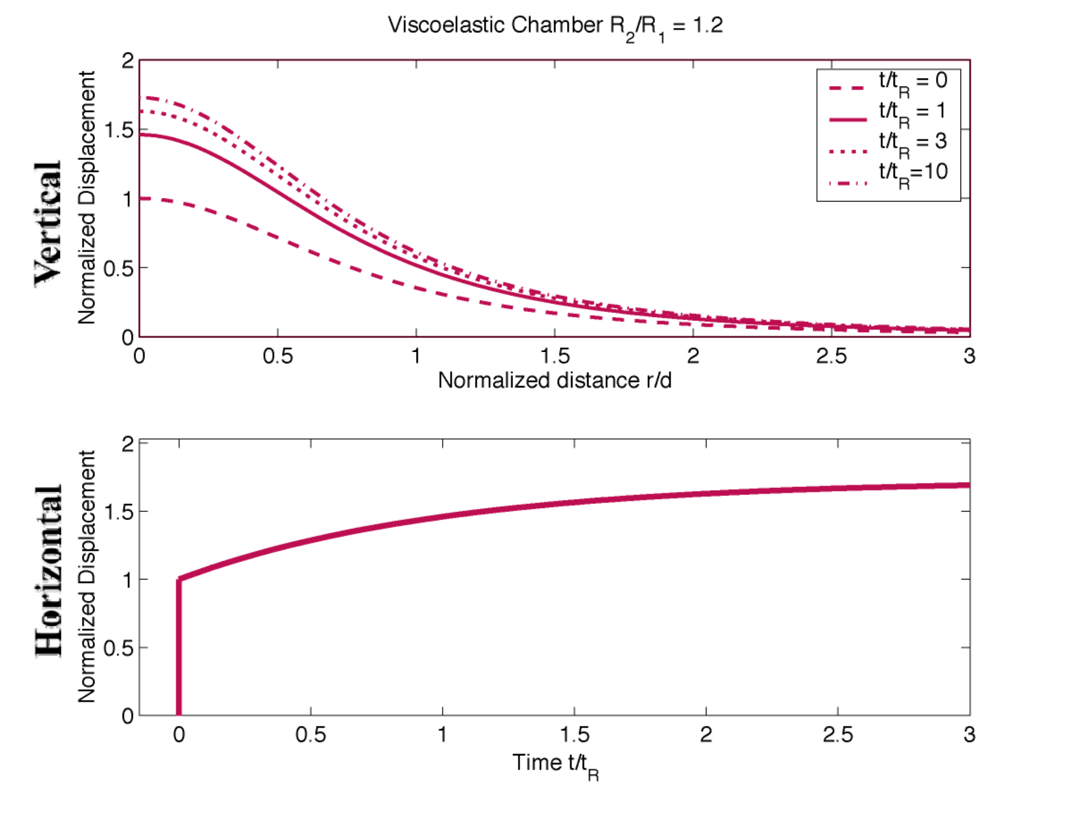
D. Dzurisin, 2007

Courtesy of M. Lisowski

Forward model

A complex example:
viscoelastic shell surrounding magma chamber





Courtesy of P. Segall

Deformation Source Models

Simple Source Models in Elastic Half-Space

- Spherical Point Source
- Prolate Ellipsoid
- Sill or Dike for volcanoes
- Penny-shaped Sill
- Pipe
- Dislocation for earthquakes

Complicating Effects

- Non-uniform Elastic Structure
- Topography
- Viscoelasticity
- Poroelasticity
- Thermoelasticity
- Complex Geometry
- Influence of hydrothermal fluid

$\underline{\mathbf{u}} = \boldsymbol{f}($*model parameters, material properties, …,* $)$

Ultimate Goal of Deformation Modeling

Minimize

$$\sum [u_i(x,y) \bullet los_i(x,y) - obs_i(x,y)]$$

u_i is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)
los_i is the InSAR line-of-sight vector
obs_i is the observed deformation (InSAR image)
(x, y) is the image coordinate

Non-linear inversion!!!!

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Find best-fit model parameters

1. loop through model parameters
   - calculate the residual (observed – modeled) for each set of model parameters

2. find the set of model parameters that renders the smallest residual
   => best-fit model parameters

A simple matlab code for deformation modeling

% **<u>Mogi_modeling.m</u>**

% define upper bounds of source parameters

ub = [21.6 23.2 7.0 -0.03 50 25 25];

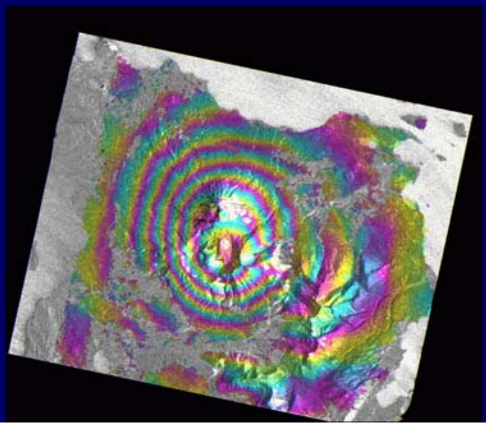
X **Y** **Z** **ΔV** static term (or baseline_error_terms)

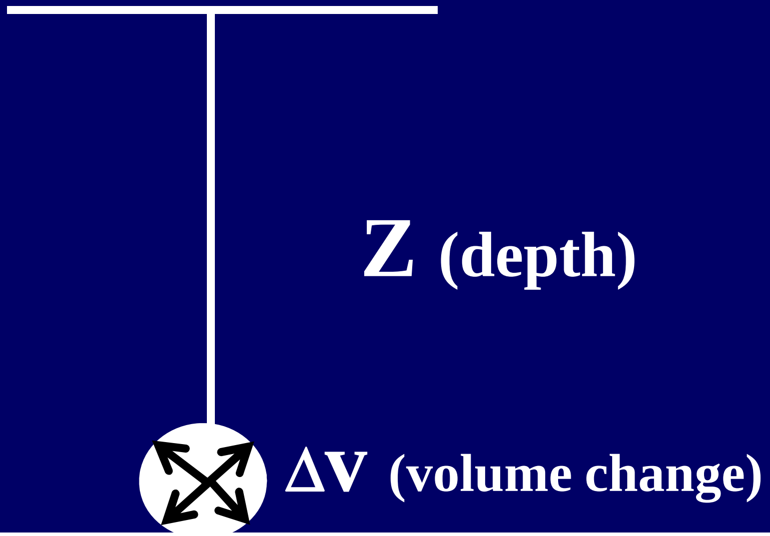
% define lower bounds of source parameters

lb = [19.6 11.2 2.0 -0.08 -50 -25 -25];

% **READ** “InSAR image and InSAR geometry parameters “

SIMULATIONS = 10;





A simple matlab code for deformation modeling

% **Mogi_modeling.m** (cont'd)

```
for i=1:SIMULATIONS
    % generate random numbers between 0 and 1.0;
    rand_vec=rand(1, source_parameter_length);
    diff_vec=ub - lb;
    p_start=lb + diff_vec*rand_vec;
    [p_new, RESNORM, residual, EXITFLAG]=…
                    lsqnonlin('mogi_func', p_start, lb, ub, opts_in);
end
```

% LSQNONLIN solves non-linear least squares problems.

% LSQNONLIN attempts to solve problems of the form:
min sum {FUN(X).^2}
% where X and the values returned by FUN (new X) can be vectors or matrices.

A simple matlab code for deformation modeling

```
% mogi_func.m
function [residual] = mogi_func(X);
%   This function will return a matrix of the residual (difference between the data
%    and calculated range change).
%
%    USEAGE:  [residual] = mogi_func(X);
%         INPUT:  X  is a vector of Mogi source parameters
%         OUTPUT: residual == a vector of observed data values minus modeled.

global  eing_vec  ning_vec  obs_phase  plook
calc_phase=rngchn_mogi(X(2),X(1),X(3),X(4), ning_vec,eing_vec,plook);

residual= obs_phase – calc_phase +X(5);
```

forward model

A simple matlab code for deformation modeling

% **rngchn_mogi.m** (forward model)

- function [rng_change]=rngchn_mogi(n1,e1,depth,del_v,ning,eing,plook);
- % USEAGE: [rng_change]=rngchn_mogi(n1,e1,depth,del_v,ning,eing,plook);
- % INPUT:
- % n1 = local north coord of center of Mogi source (km)
- % e1 = local east coord of center of Mogi source (km)
- % depth = depth of Mogi source (km).
- % del_v = Volume change of Mogi source (km^3)
- % ning = north coord's of points to calculate range change
- % eing = east coord's of points to calculate range change
- % OUTPUT: rng_change = range change at coordinates given in ning and eing.

$$u_i(x_1 - x_1', x_2 - x_2', -x_3) = \Delta V \frac{(1-\nu)}{\pi} \frac{x_i - x_i'}{\left|R^3\right|}$$

forward model

Multiple Sources

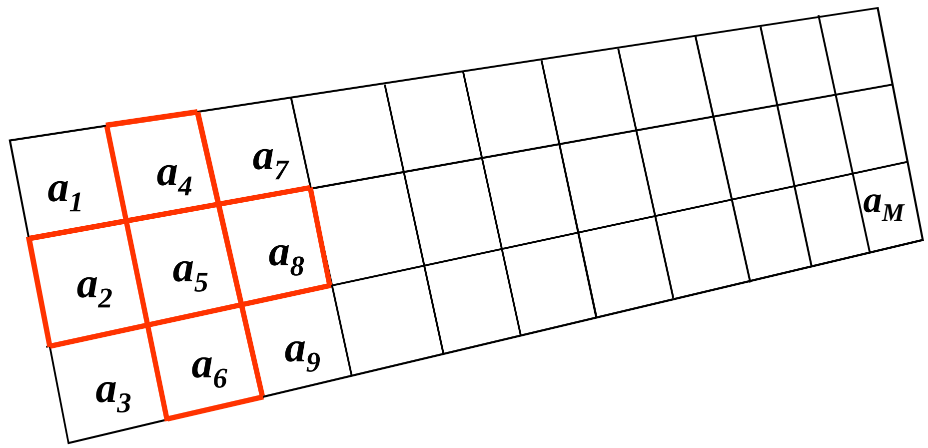
- Superimposition of individual deformation sources
- Smoothing (spatial + temporal)

Spatial smoothing

- The total displacement on a given patch…
- …is related to that of patches adjacent to it, by a finite-difference Laplacian approximation

$$(a_2-a_5)-(a_5-a_8)+(a_4-a_5)-(a_5-a_6) = 0$$

$$a_2 + a_4 - 4a_5 + a_6 + a_8 = 0$$



(schematic)

Courtesy of G. Funning

Source parameter error estimates

•**One approach of estimating parameter errors is Monte Carlo simulation of correlated noise (**Wright, Lu & Wicks, 2003**).**

•**Multiple sets of correlated noise are simulated that have the same covariance function as observed in the data.**

•**A number of such data sets are added to the observation (e.g., InSAR phase changes).**

•**Parameter errors are determined from the distribution of the best-fit solutions to each of these noisy data sets.**

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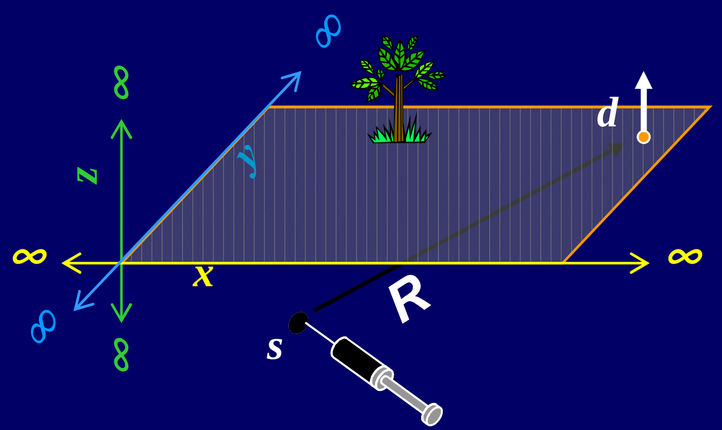
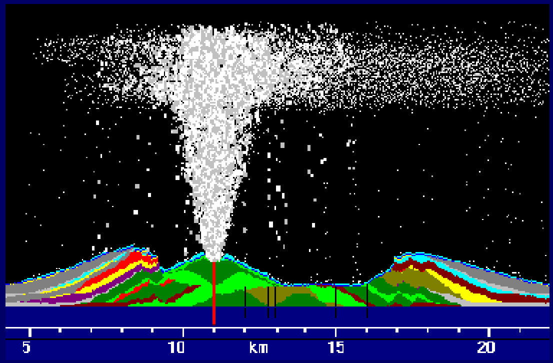
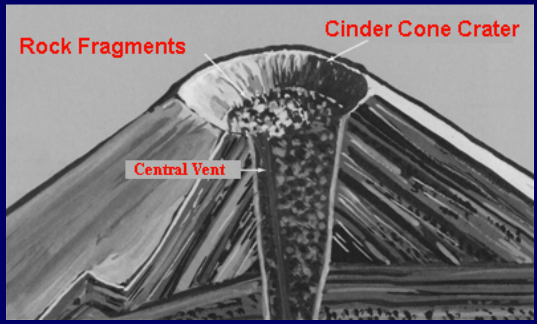
Volcano structure

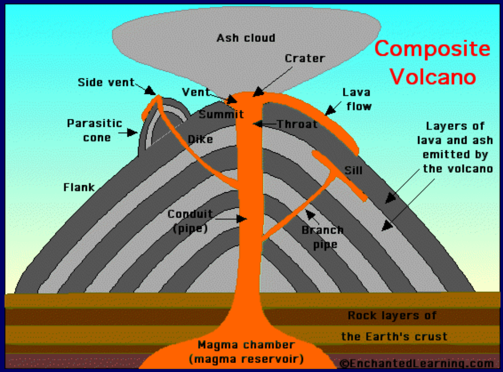
Basic concepts

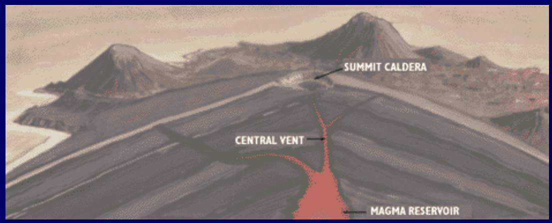
standard model





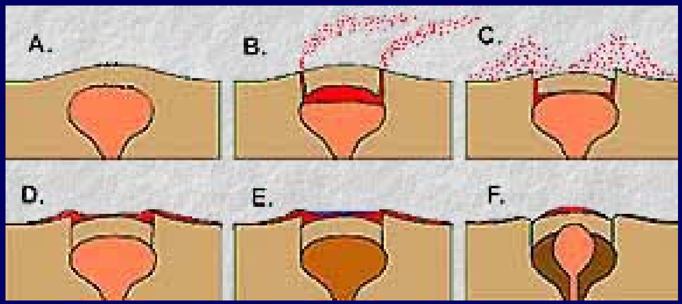






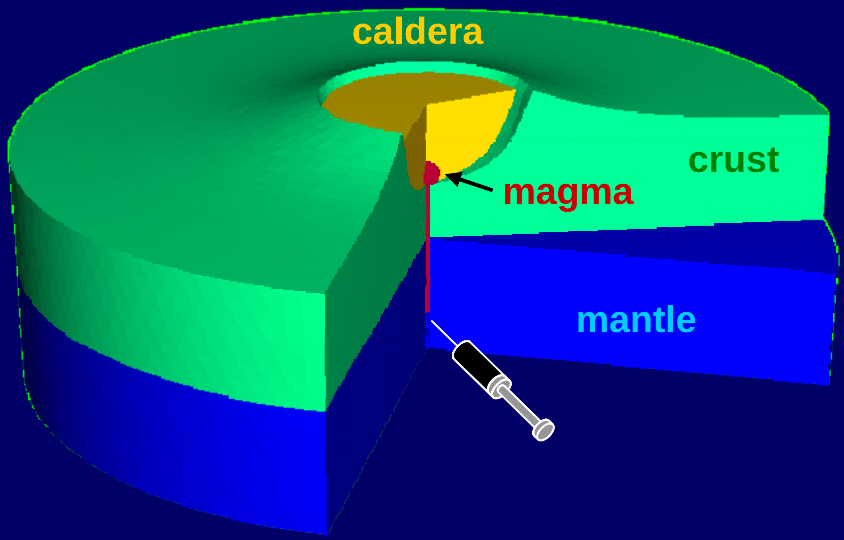
<u>required</u> assumptions:

- ***homogeneous material properties***
- ***isotropic material properties***
- ***Poisson-solid***
- ***half-space***



Finite element models

Simulate volcano structures



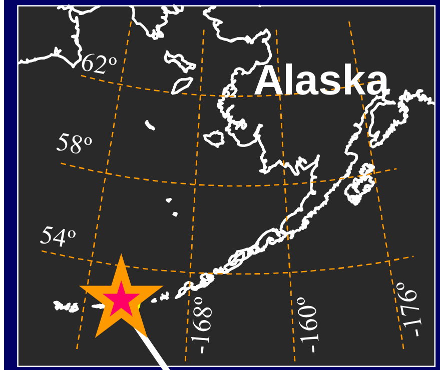
elasto-static behavior

$$\mu \nabla^2 u_i + \frac{\mu}{(1-2\upsilon)} \left[\frac{\partial^2 u_k}{\partial x_i \partial y_k} \right] = -F_i$$

Courtesy of T. Masterlark

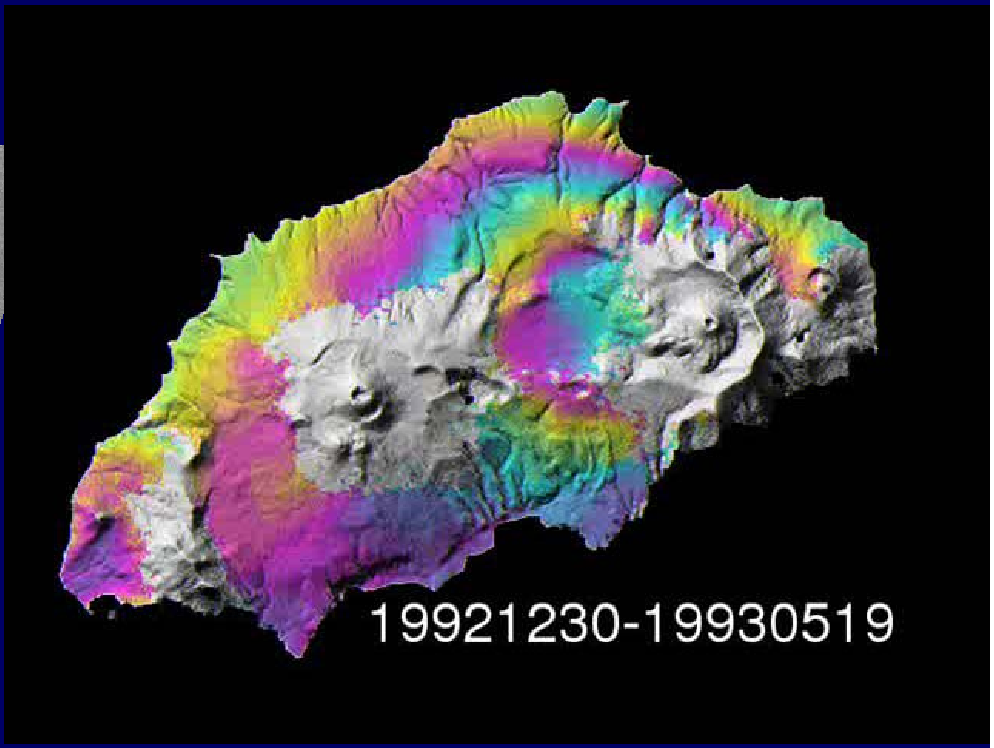
Example 1

Dynamic deformation of Seguam volcano



Seguam Volcano: Documented eruptions occurred in 1786-1790, 1827, 1891, 1892, 1901, 1927, 1977, and 1992-1993.

Multi-temporal InSAR Images

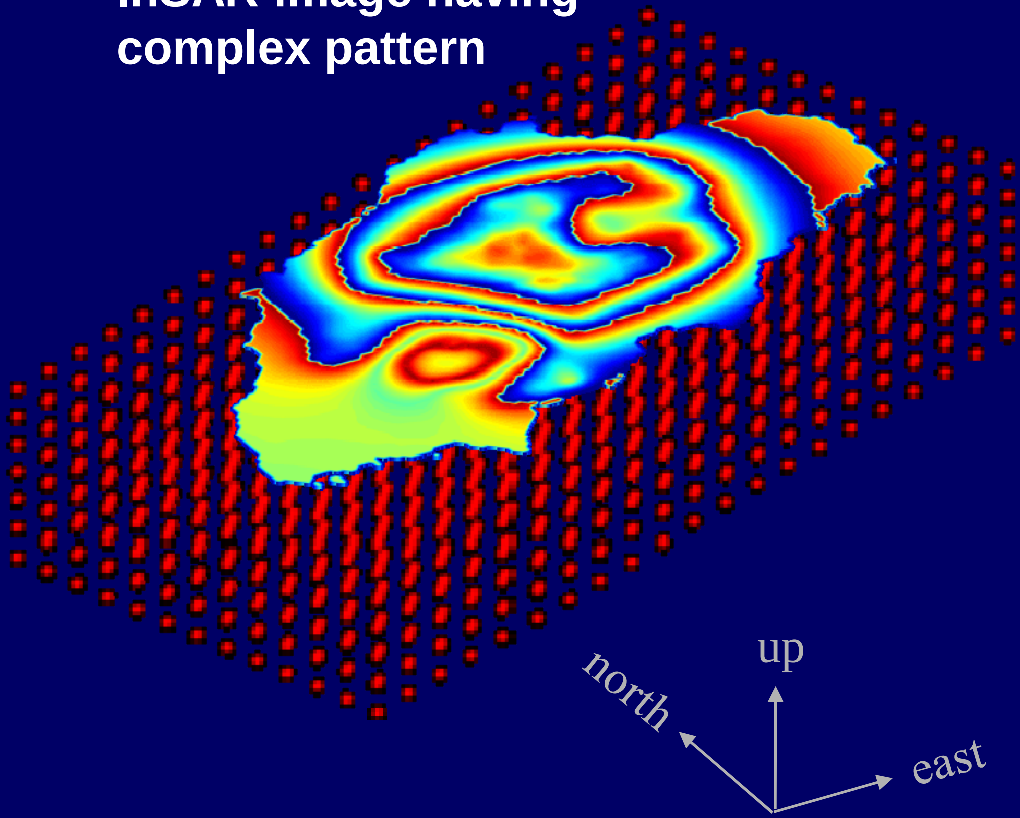


Masterlark & Lu, 2004

Deformation Modeling

point expansion source array

InSAR image having complex pattern



$$d_j = s_i \left(\frac{-z_i}{R^3_{ij}} \right)$$

displacement

source strength

G_{ij}

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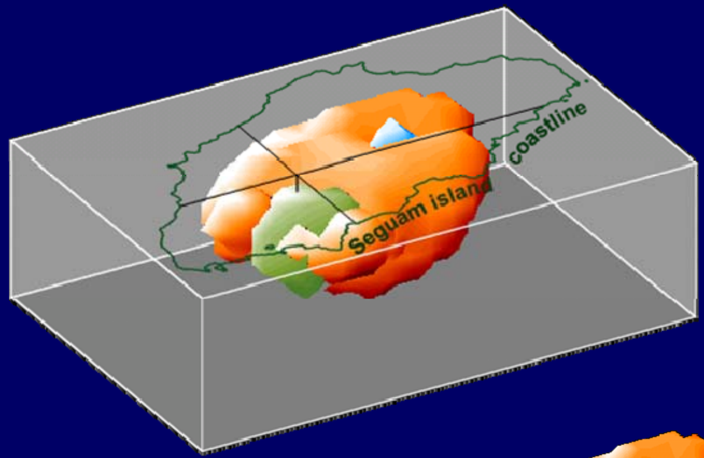
Source cluster time series

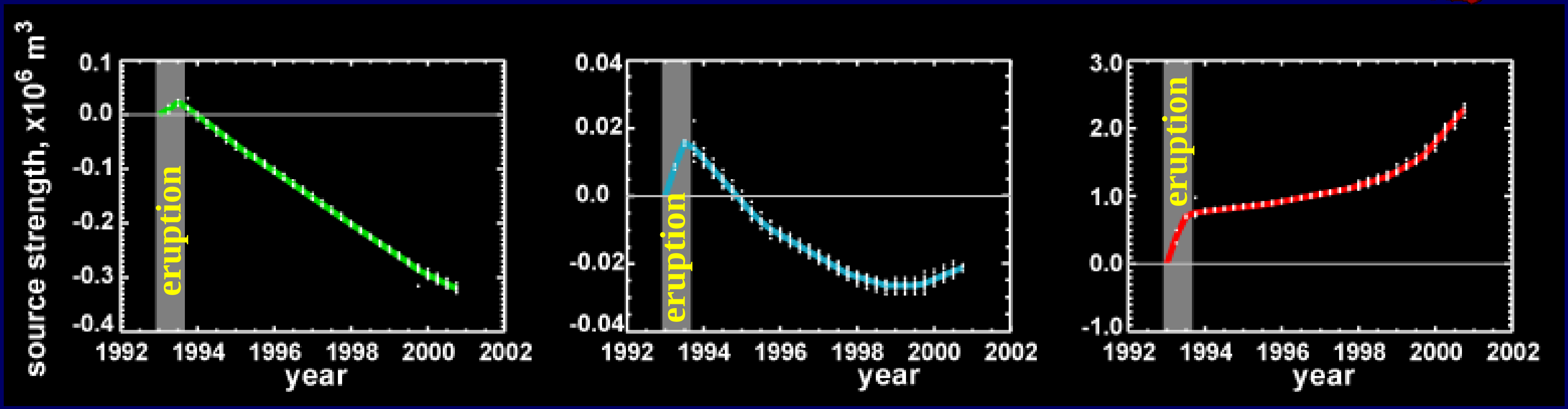
Three clusters dominate, each having a distinctive time-dependent behavior



cluster 1

cluster 2

C3

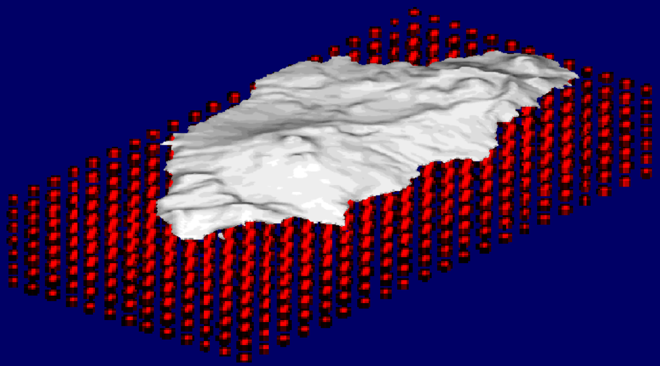


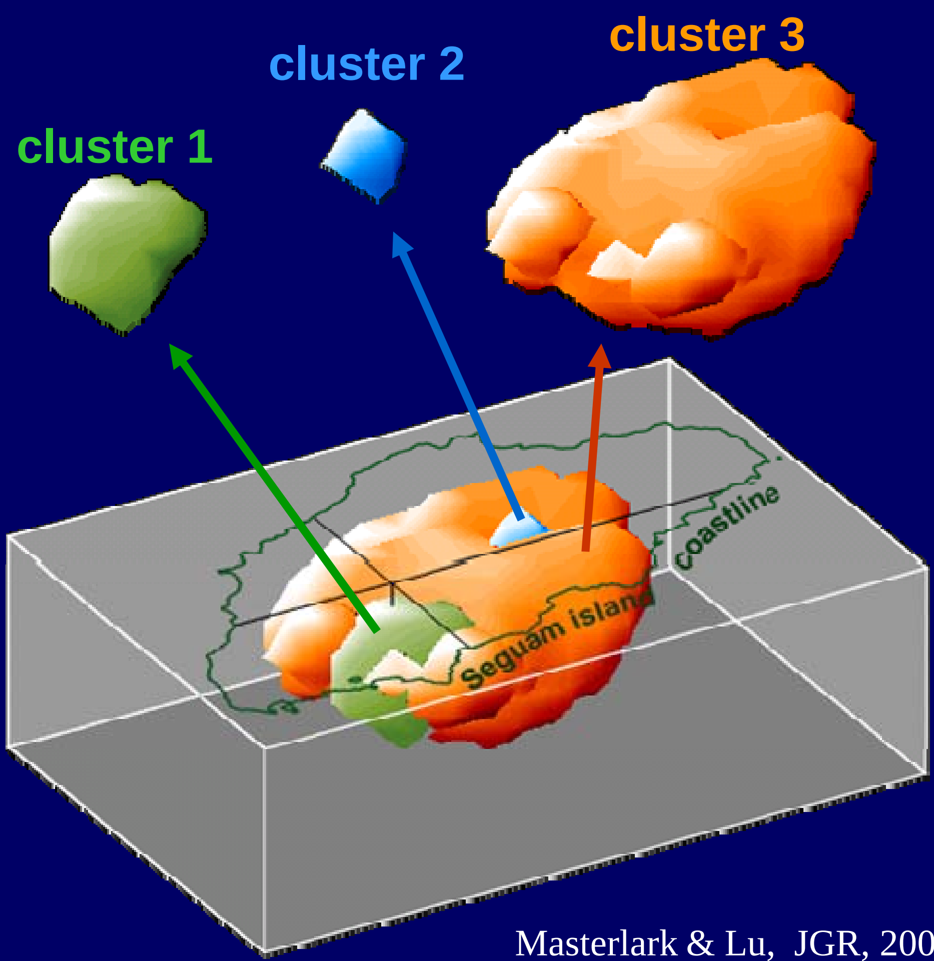
Masterlark & Lu, JGR, 2004

Dominant Source Clusters

Three clusters dominate, each having a distinctive time-dependent behavior

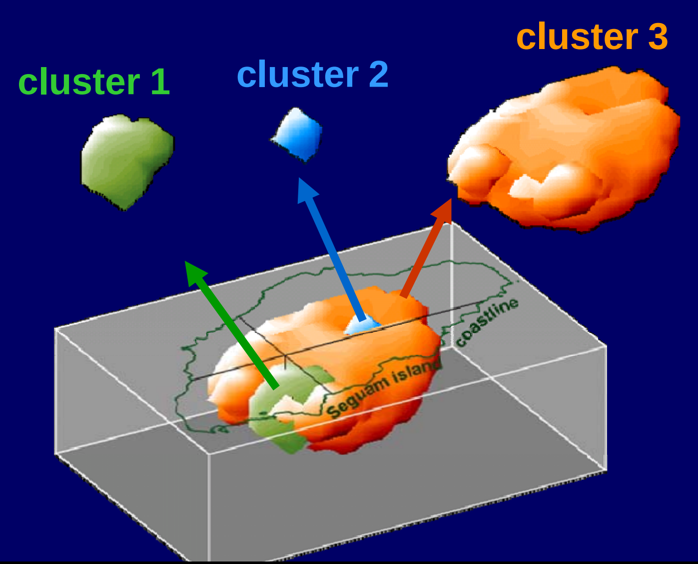
potential point sources…

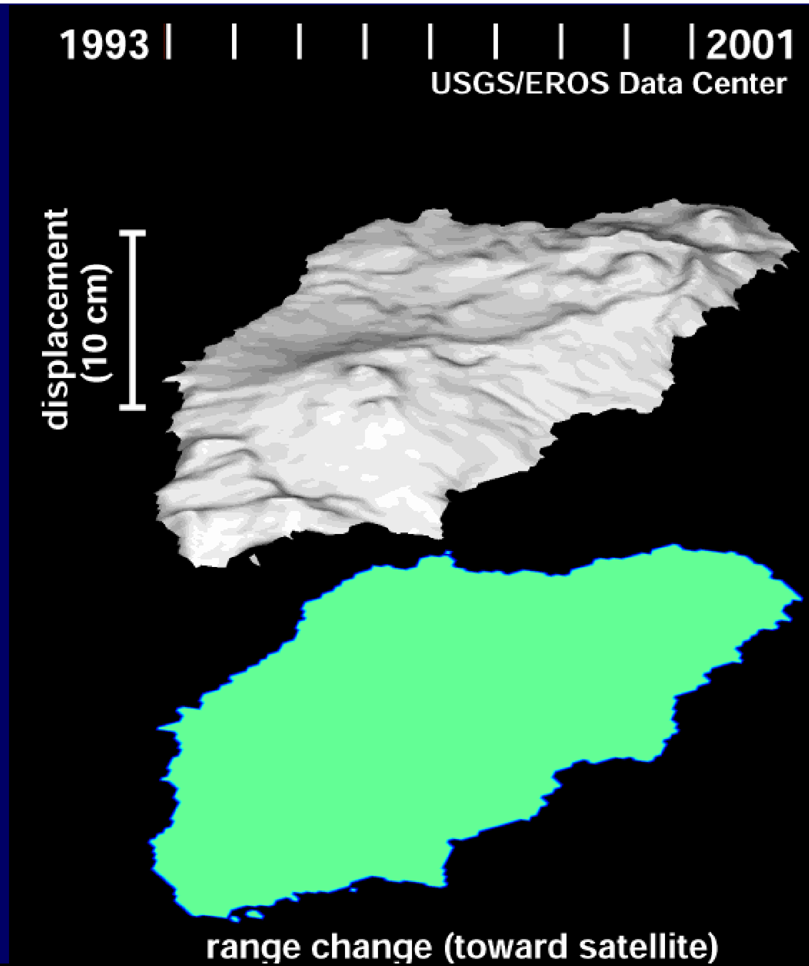


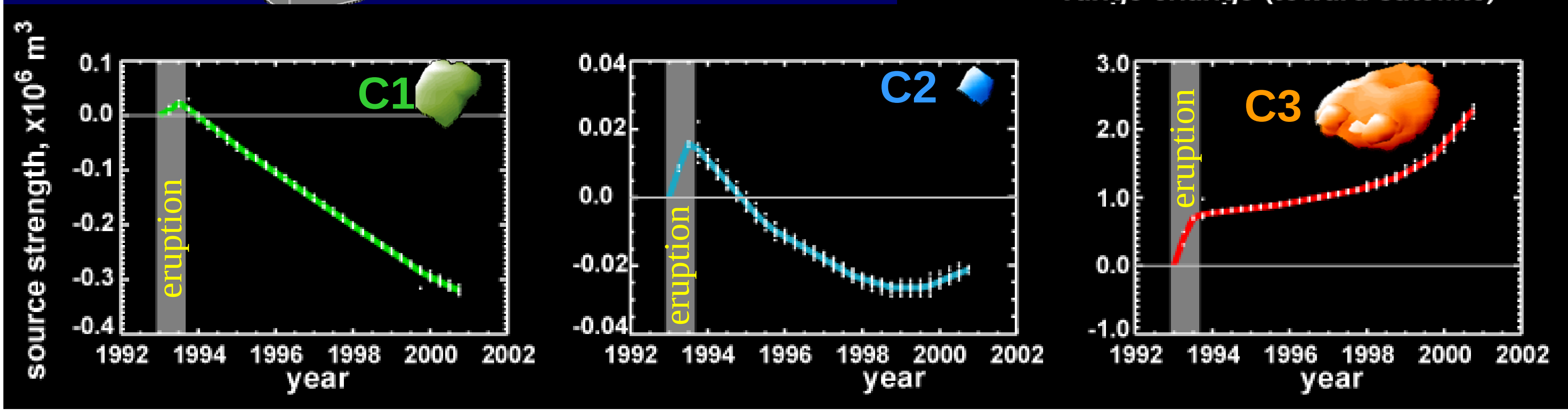


Masterlark & Lu, JGR, 2004

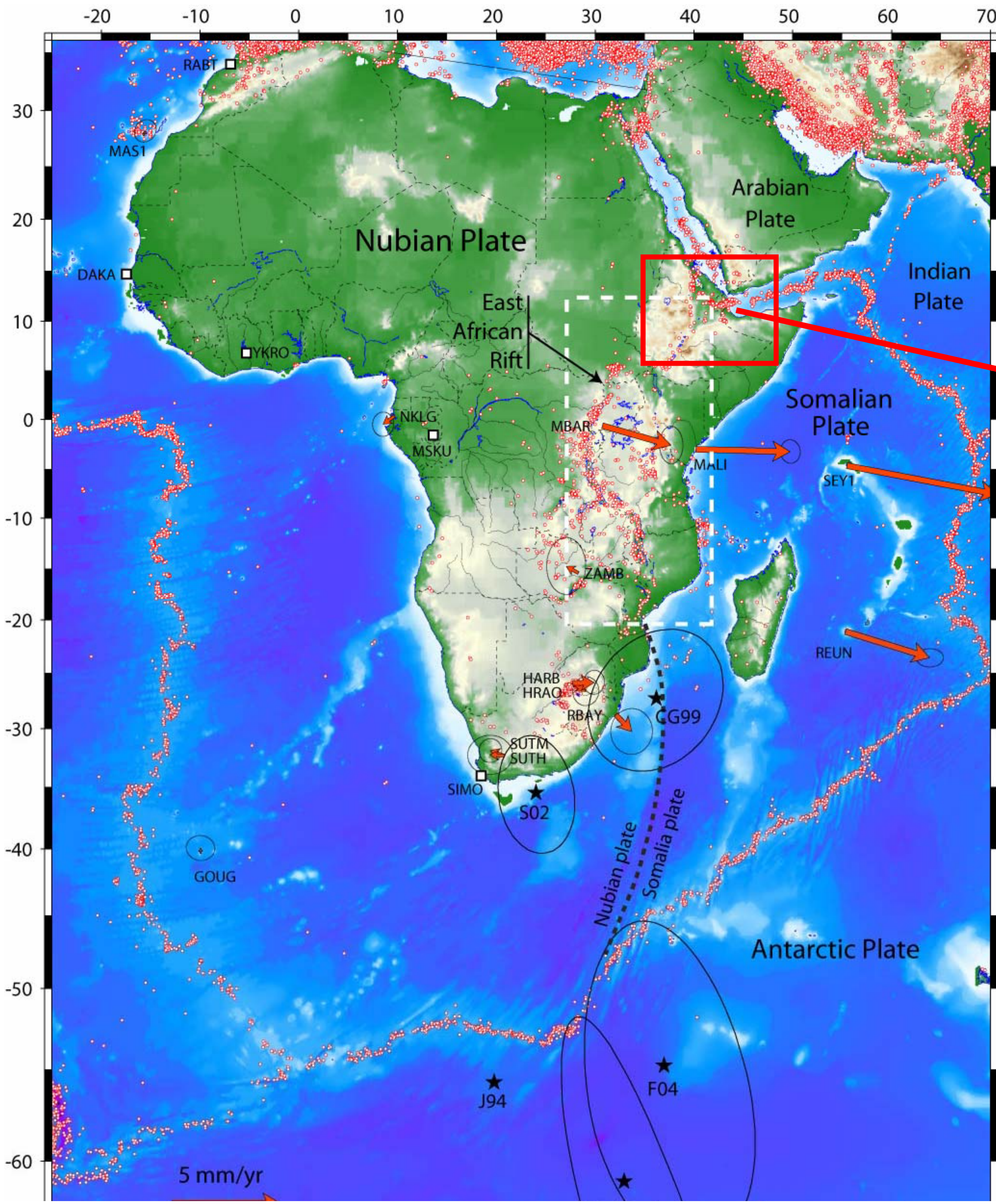
Transient deformation



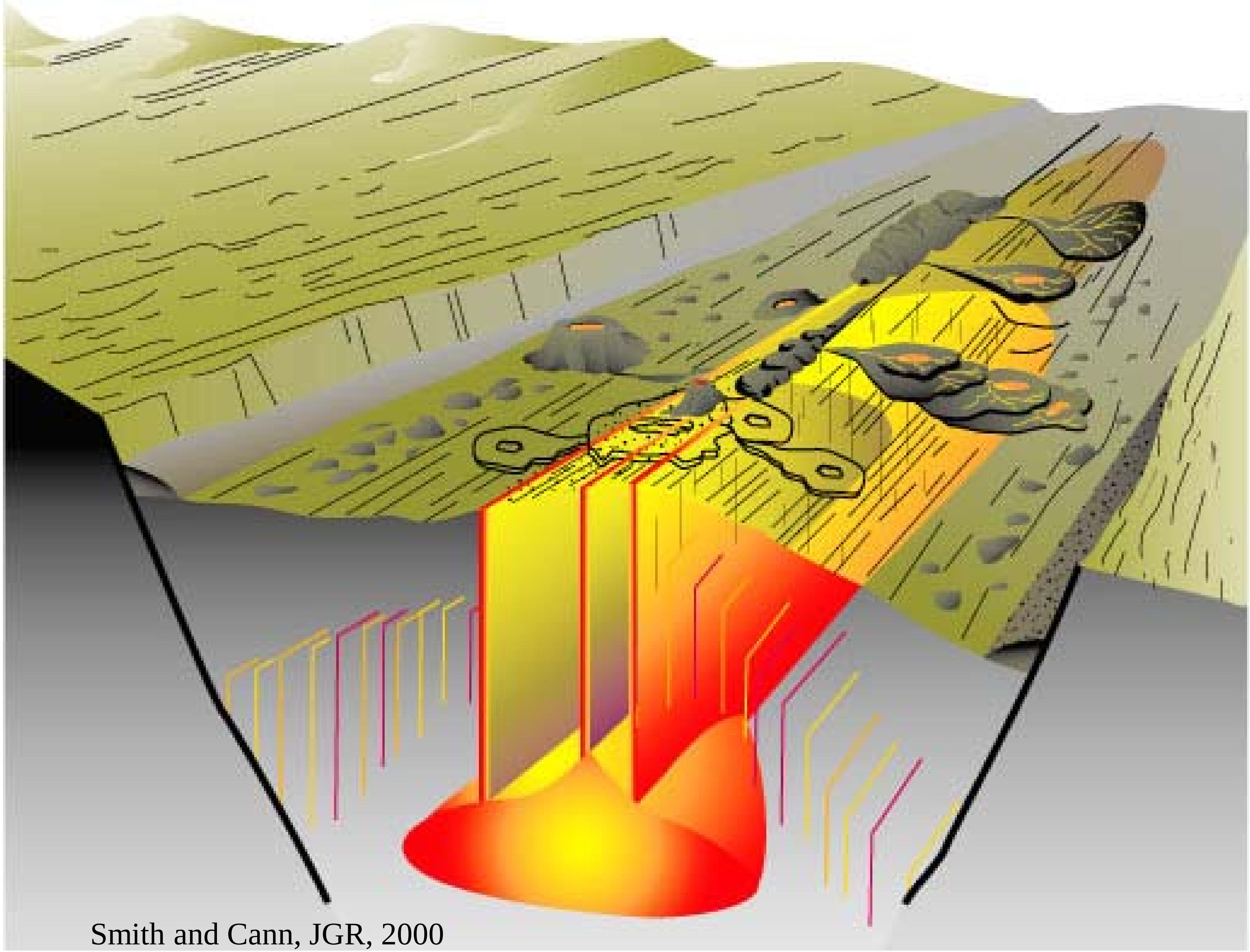




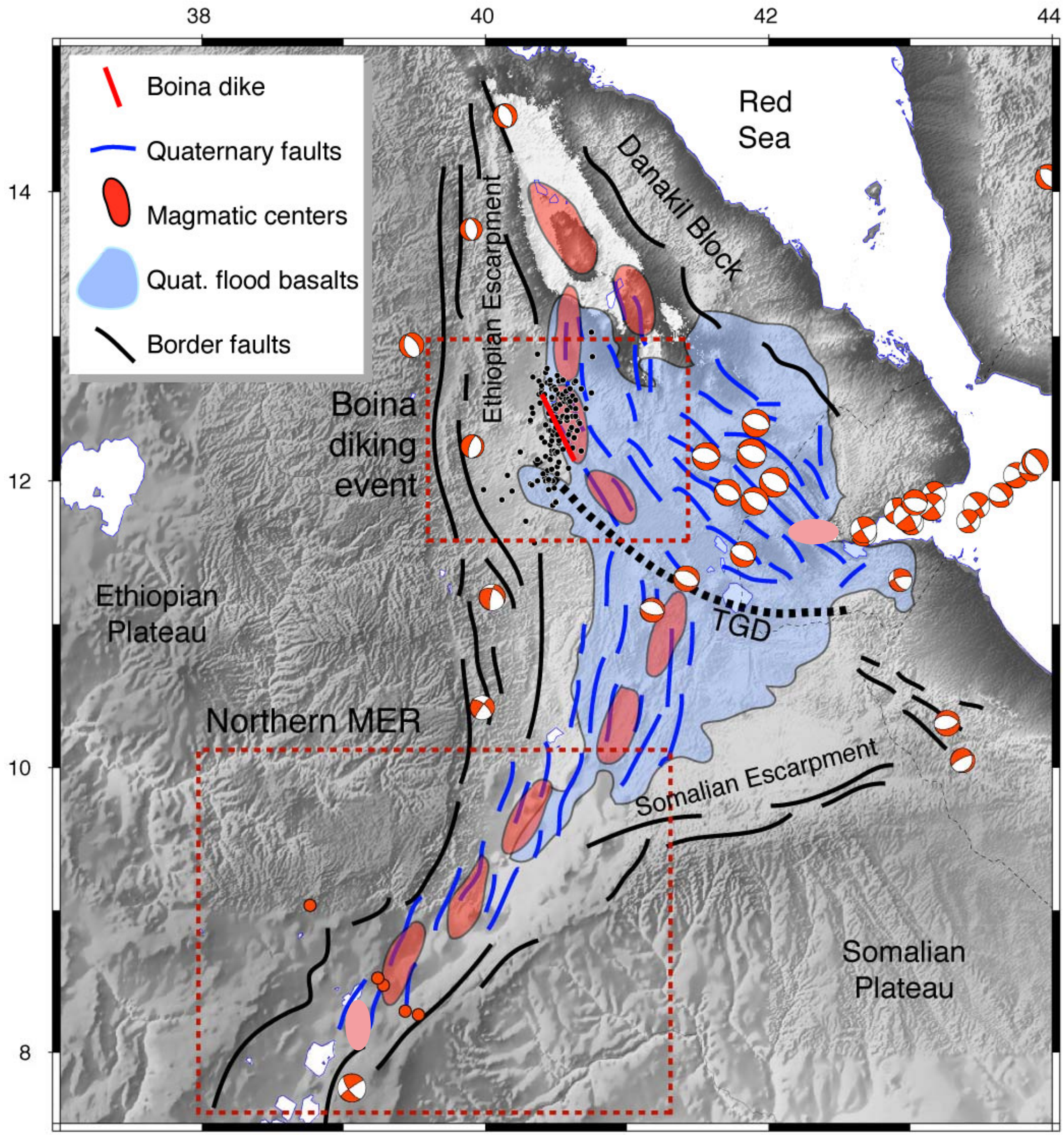
Example 2



Afar – triple junction.

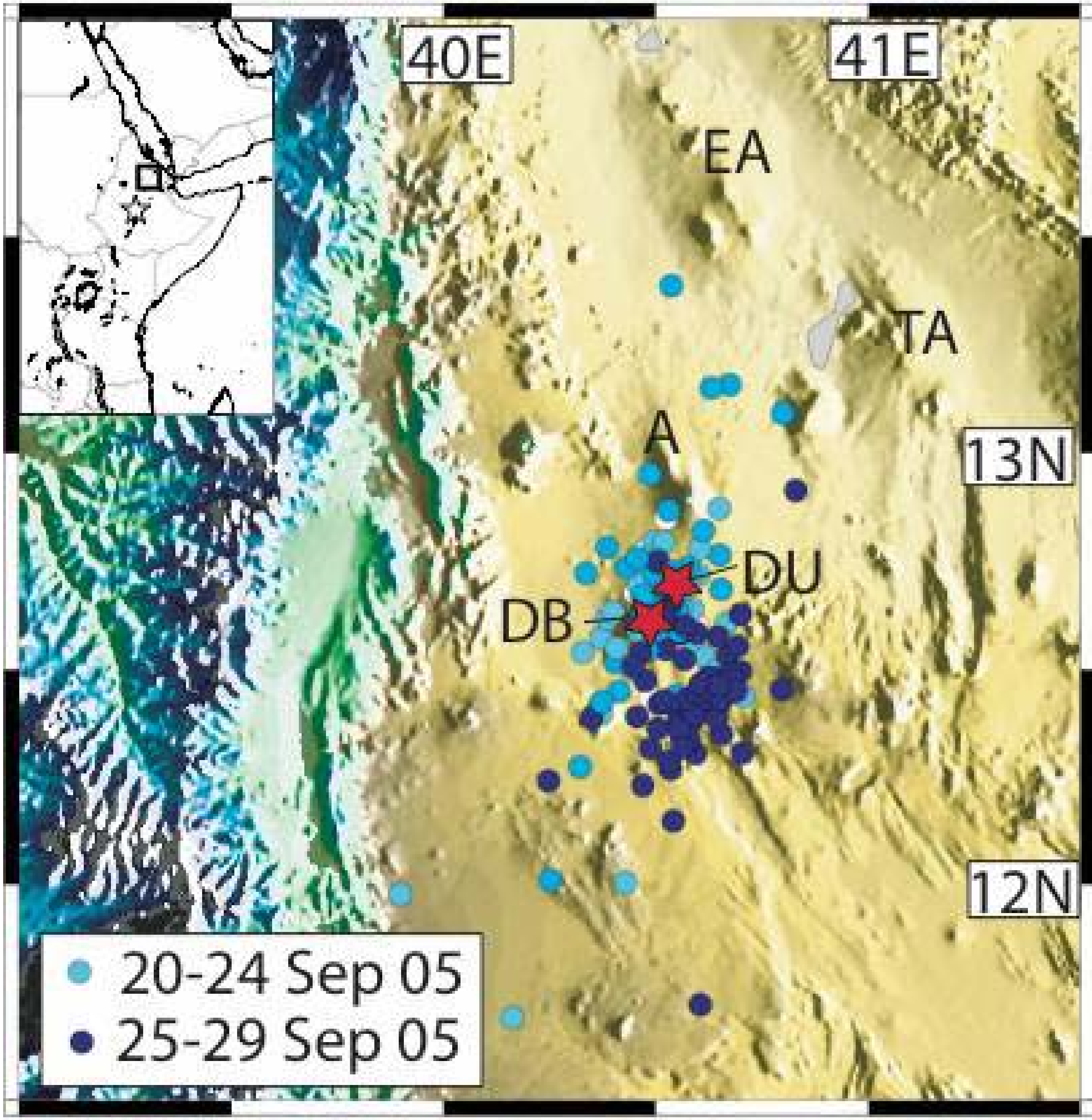


Smith and Cann, JGR, 2000



Quaternary strain localised to ~60 km long zones of fissures, aligned eruptive centers and faults - "magmatic segments "

Courtesy of T. Wright

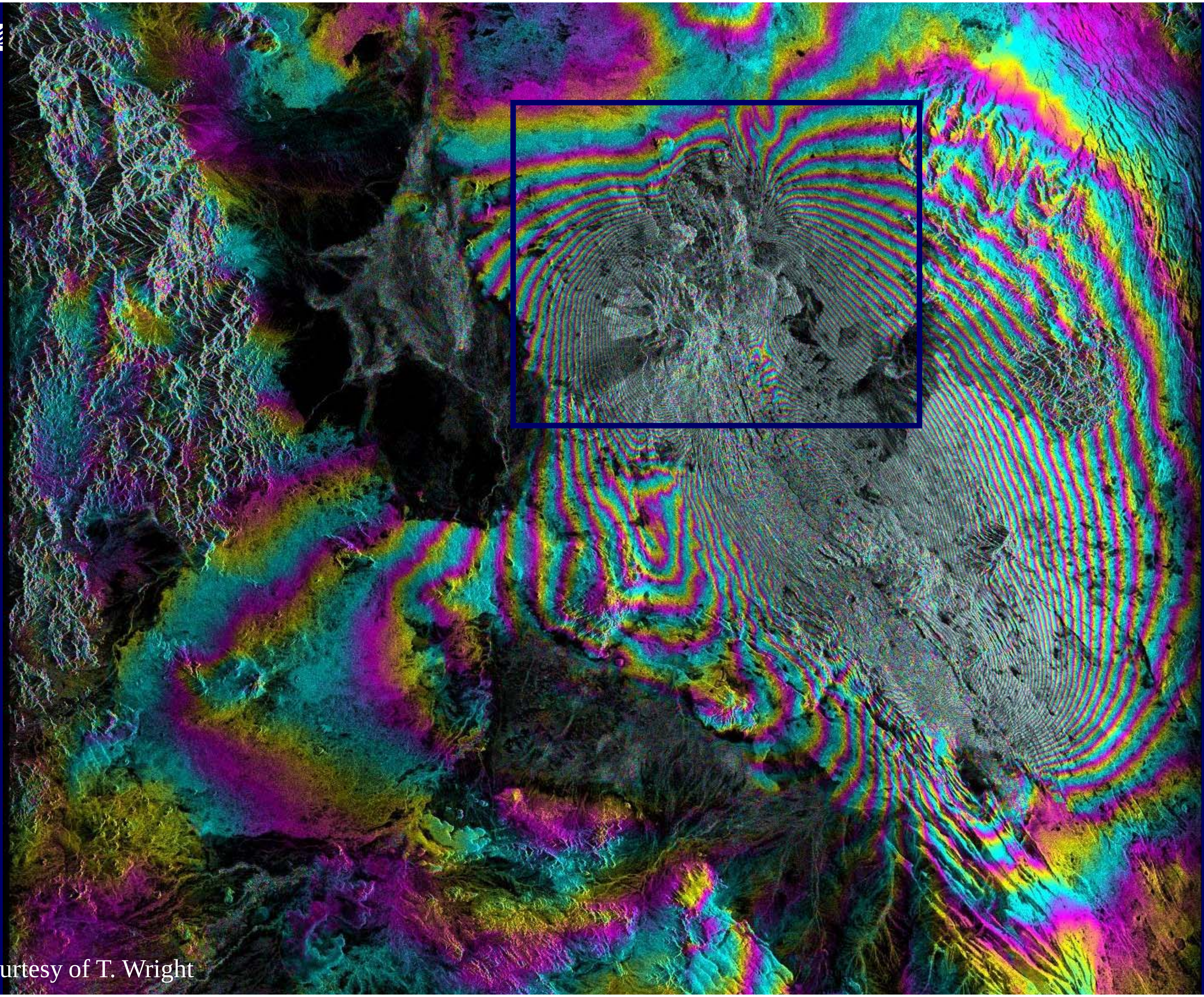


14/9/2005 to 11/05/2005

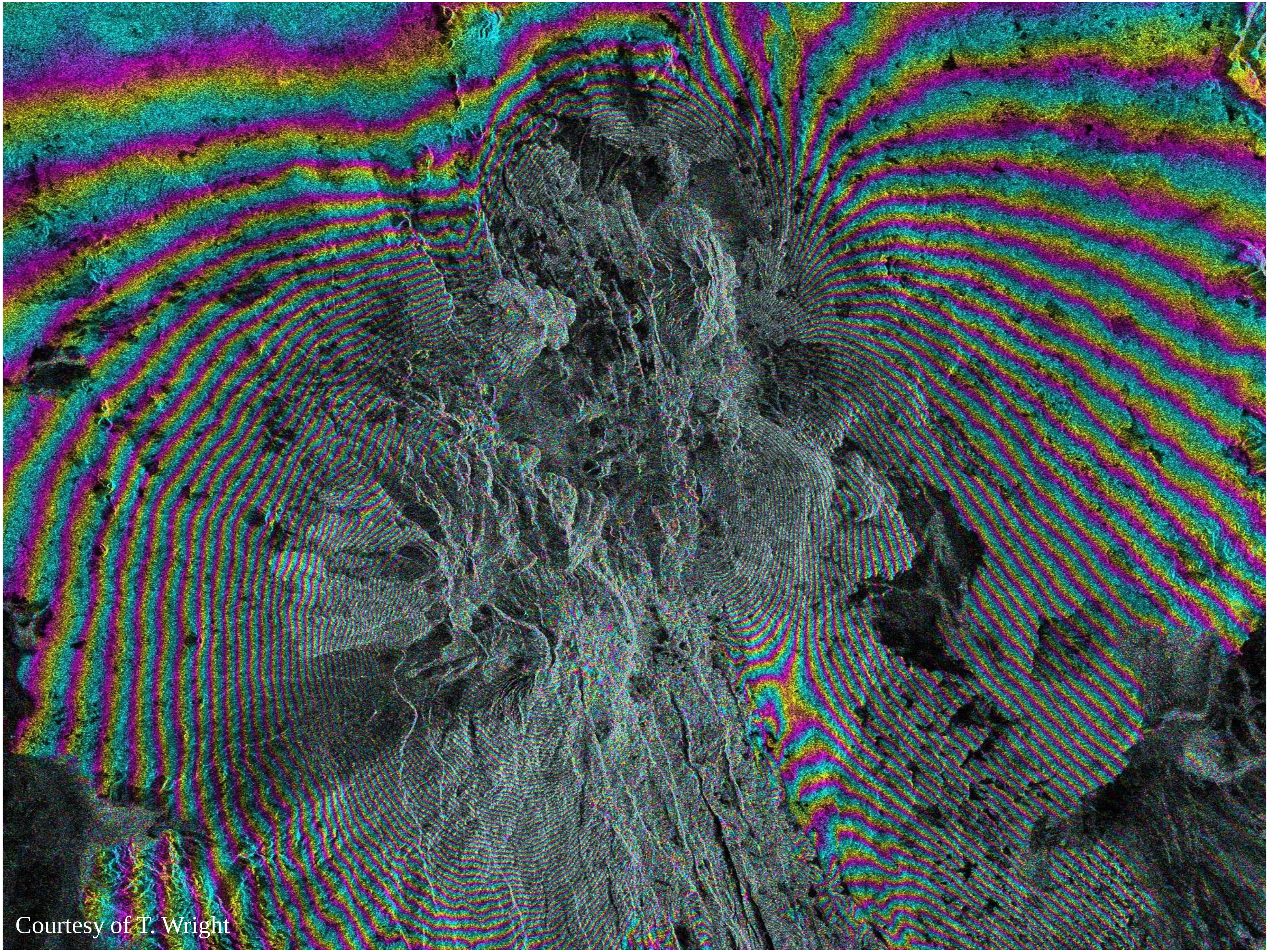
163 earthquakes (mb <6) detected by NEIC.

Relocated by Anna Stork

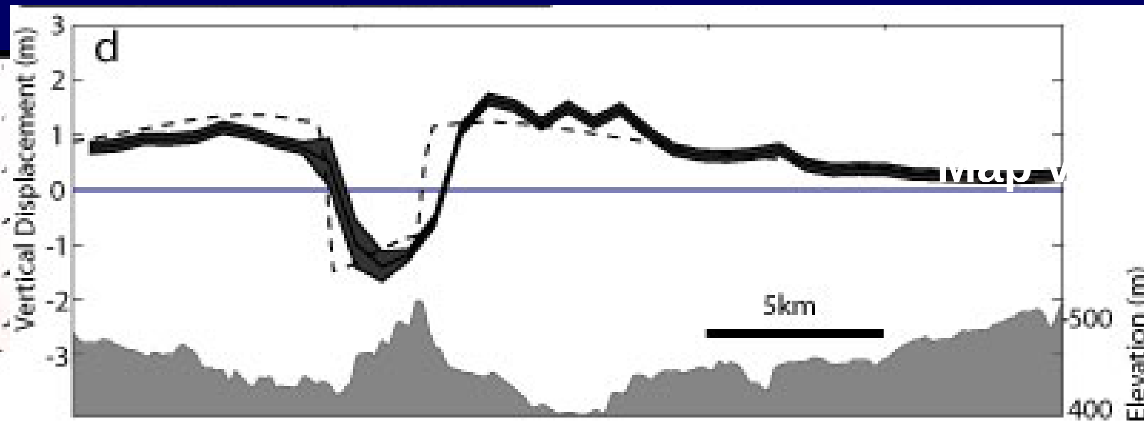
Courtesy of T. Wright



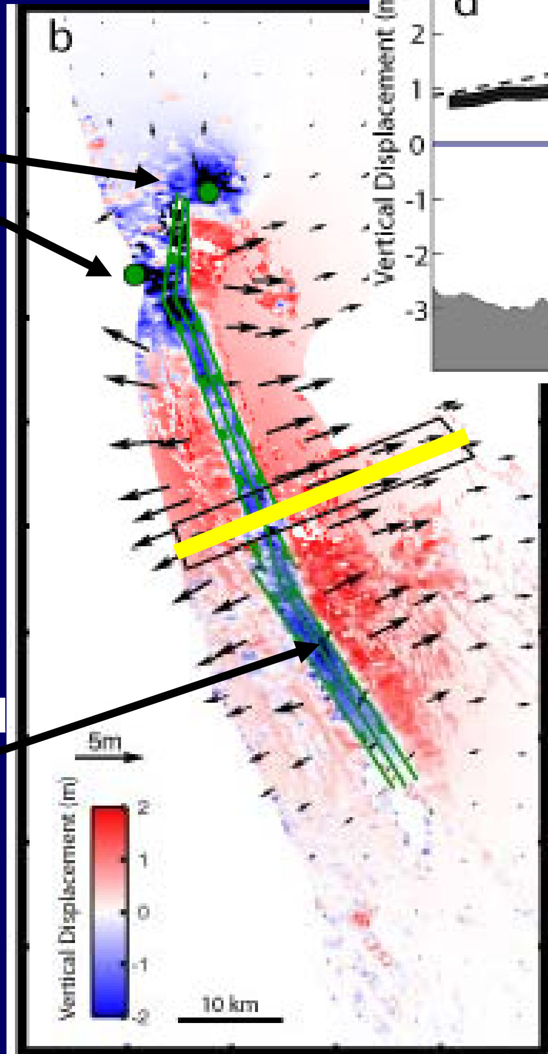
Courtesy of T. Wright


Courtesy of T. Wright

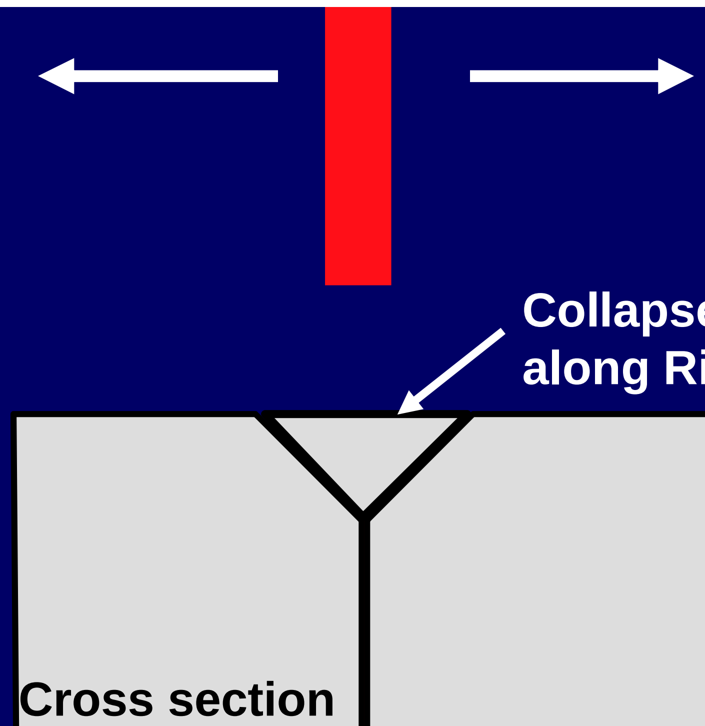
3D displacements measured from radar data



Deflating Magma Chambers

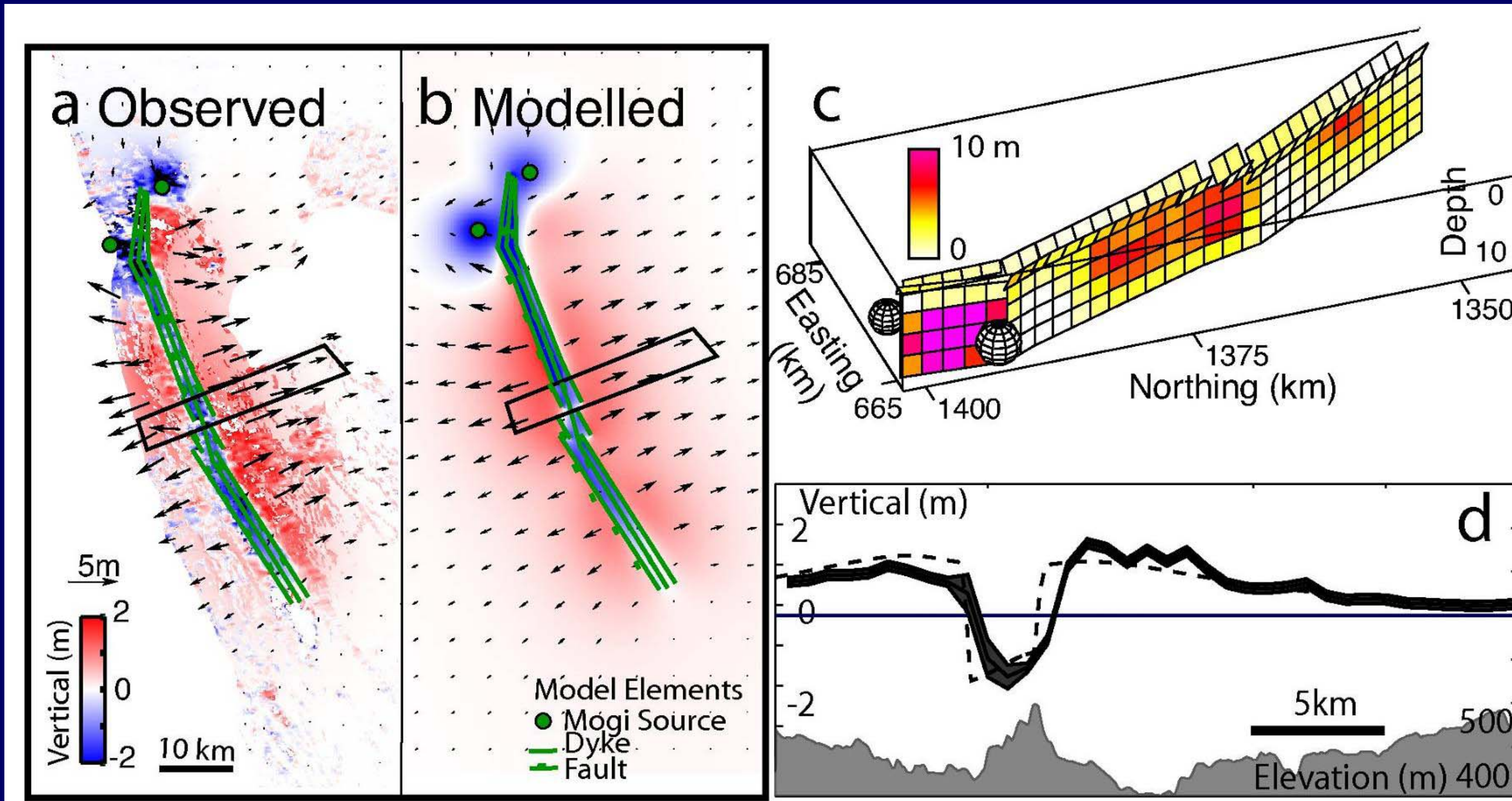


Collapsed Zone along Rift



Collapsed Zone along Rift

Deformation Modelling



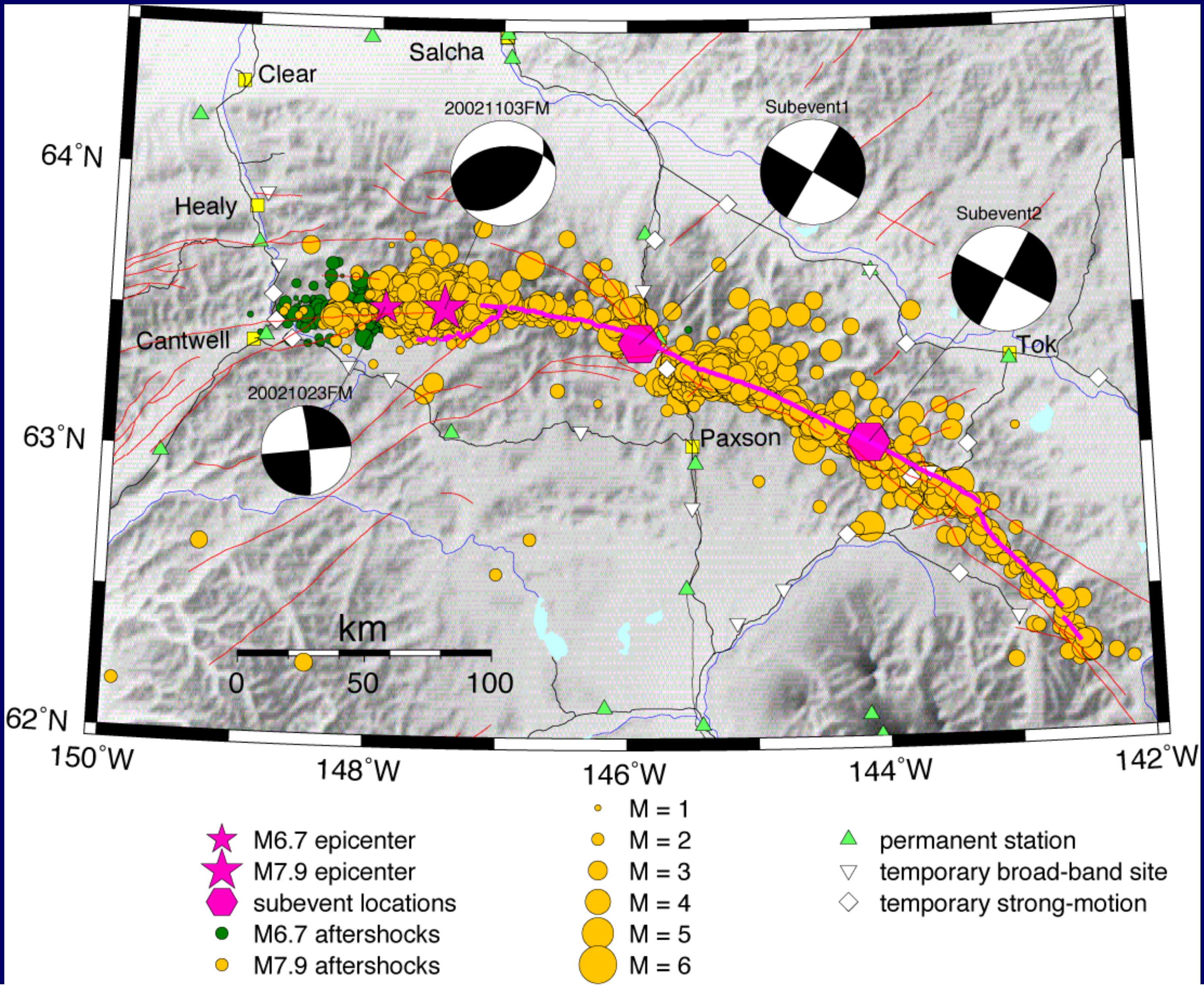
- 2.2 km^3 magma intruded along dyke (Mt St Helens 1980 1.2 km^3)
- 0.5 km^3 sourced from Dabbahu and Gabho volcanoes at North.
- Earthquakes can be responsible for < 10 % of moment release.

Wright et al., Nature, 2005

Example 3

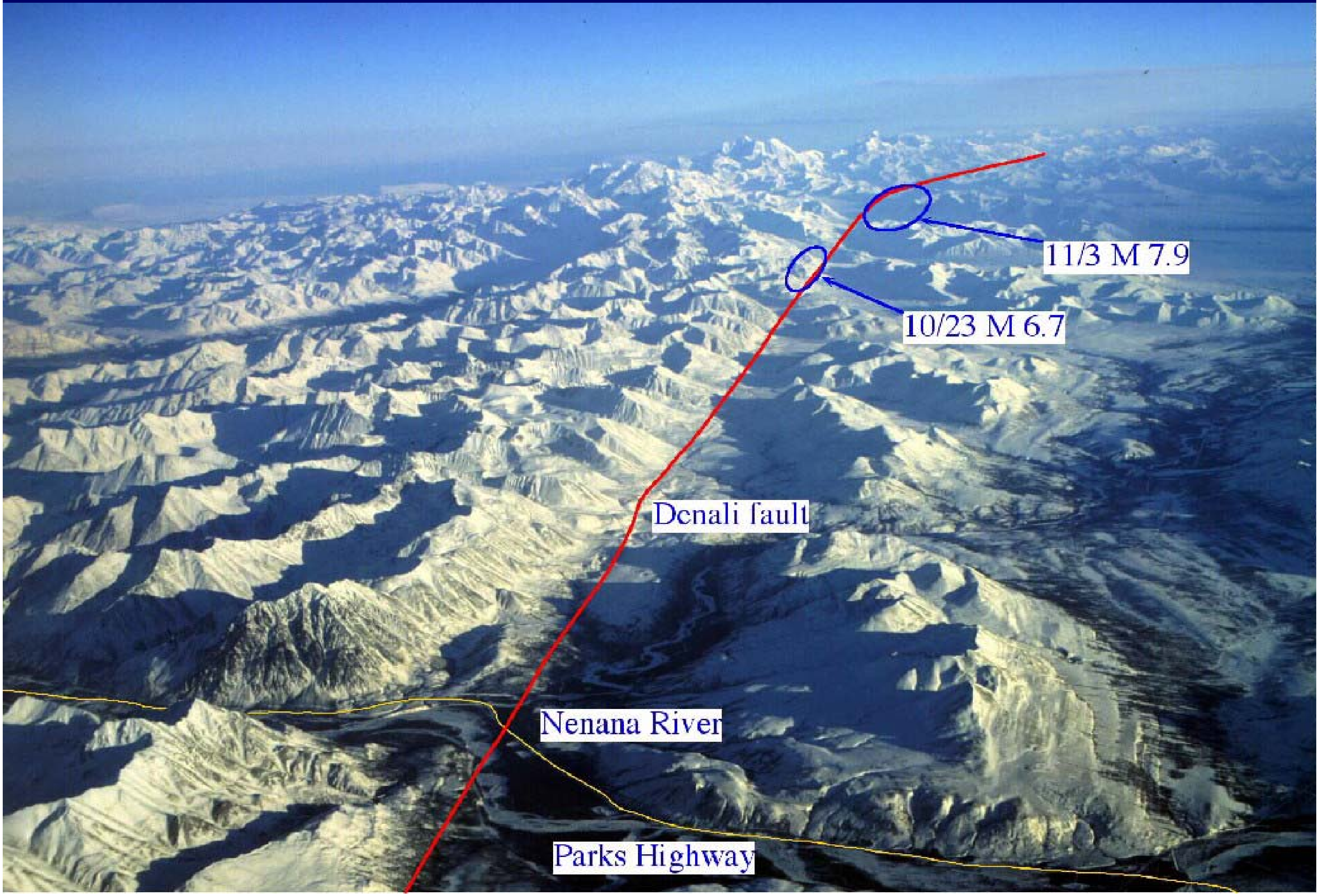
USGS

Oct. 23 and Nov 3, 2002 Denali Earthquakes



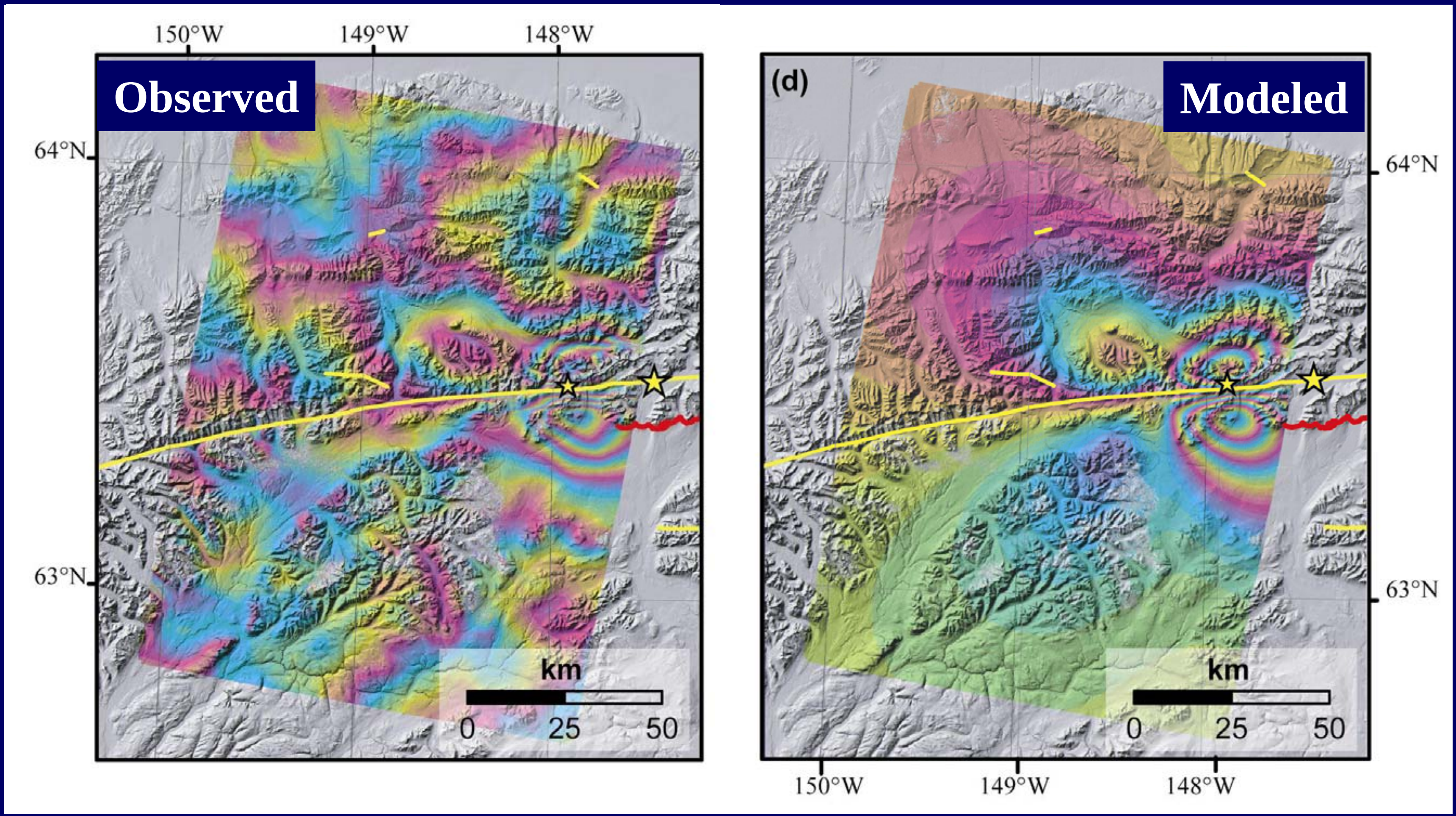
USGS

2002 Denali Fault Earthquakes



InSAR images: observed and modeled

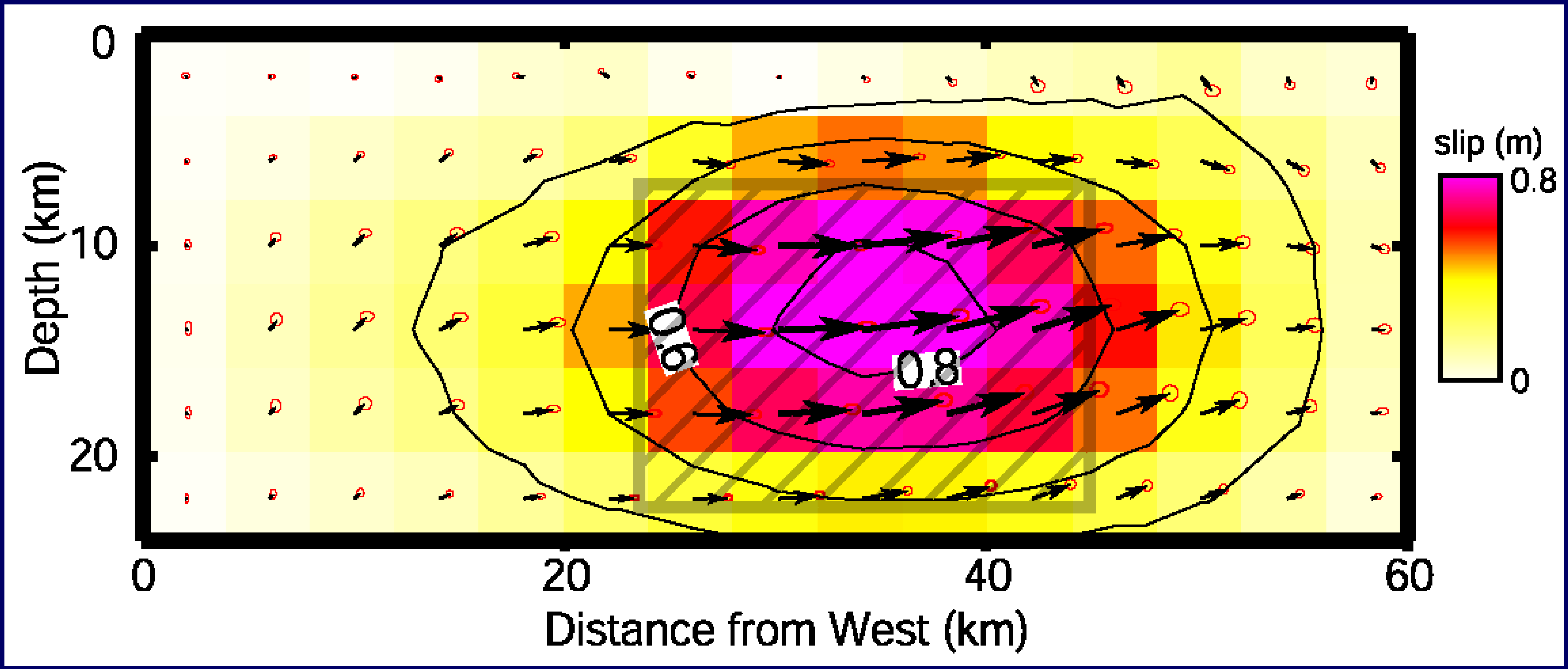
Oct. 23, 2002 Earthquake



• Lu, Wright, Wicks, EOS, 2003

USGS

Slip Distribution of Oct 23, 2002 Earthquake



• Wright, Lu, Wicks, GRL, 2003

Model Parameter Error Bounds

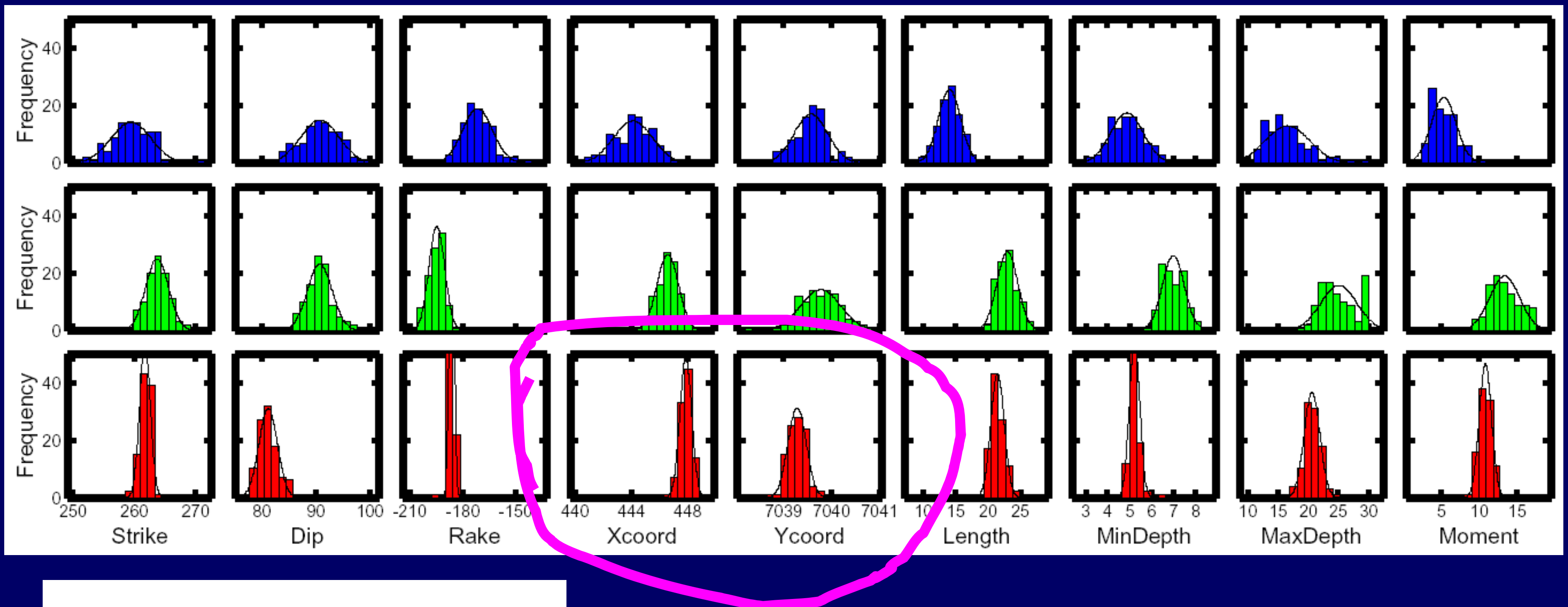
- One approach of estimating parameter errors is Monte Carlo simulation of correlated noise (Wright, Lu & Wicks, 2003).

- Multiple sets of correlated noise are simulated that have the same covariance function as observed in the data.

- A number of such data sets are added to the observation (e.g., InSAR phase changes).

- Parameter errors are determined from the distribution of the best-fit solutions to each of these noisy data sets.

Model Parameter Error Bounds



- 1 ascending interferogram
- 1 descending interferogram
- 6 interferograms (asc. & desc.)

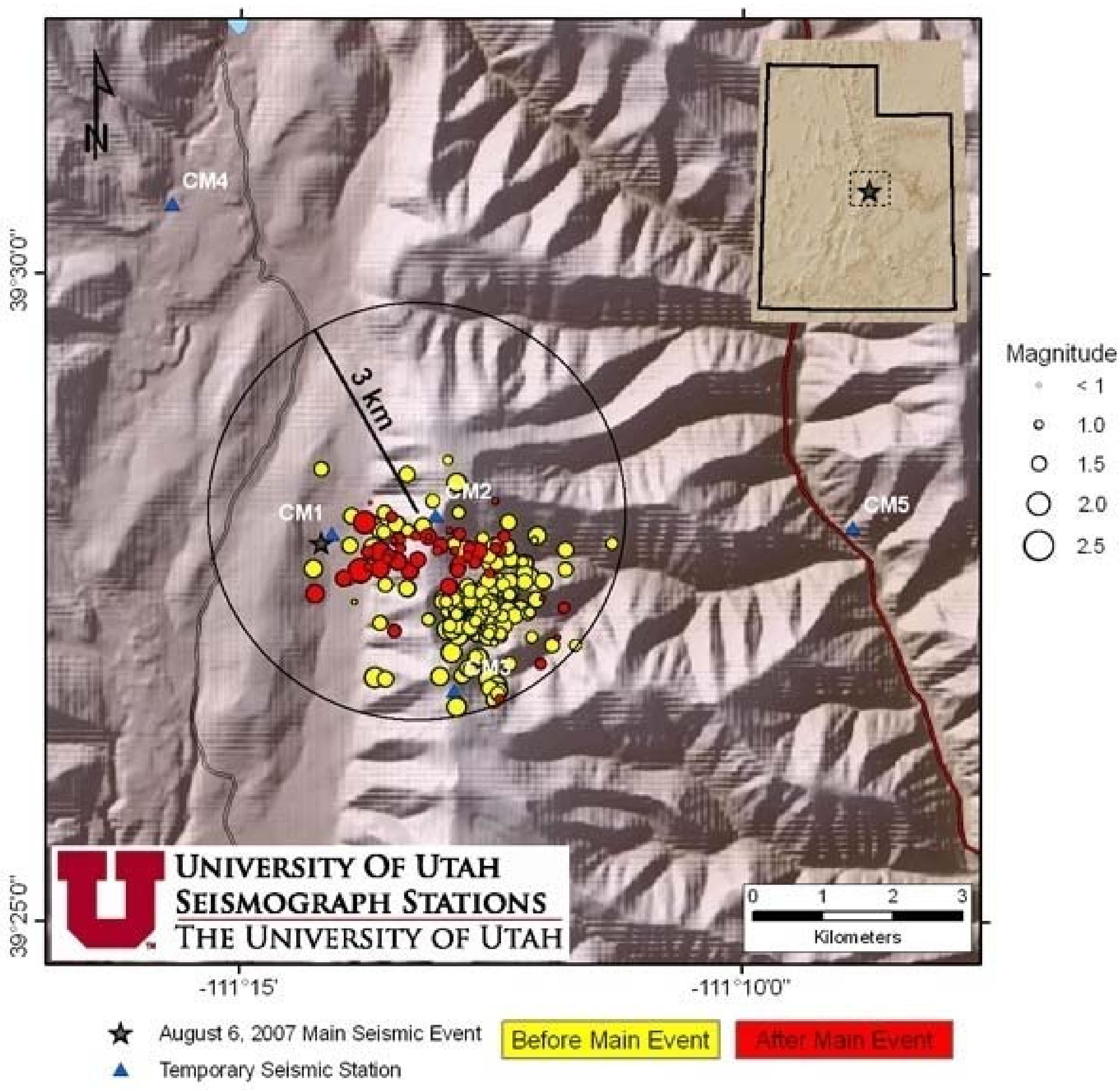
Note location errors << 1 km

Wright, Lu and Wicks, GRL 30 (18), 2003

Example 4

6 August 2007 Mine Collapse and M_L3.9 Earthquake

- A large and tragic collapse occurred in the Crandall Canyon coal mine on 6 Aug. 2007, causing the loss of 6 miners.
- This collapse was accompanied by a local magnitude (M_L) 3.9 seismic event having a location and origin time coincident with the collapse (within current uncertainty limits)



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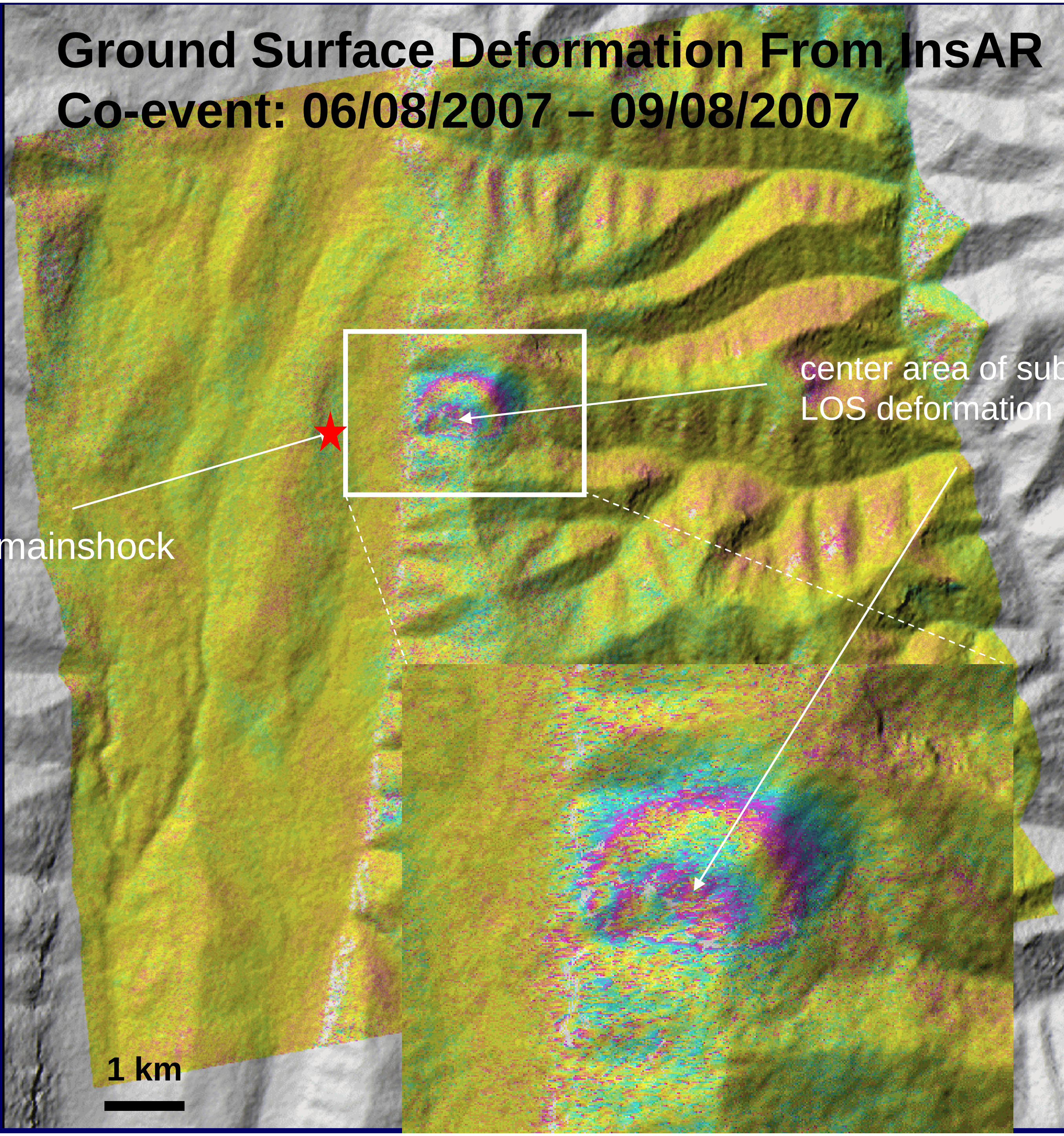
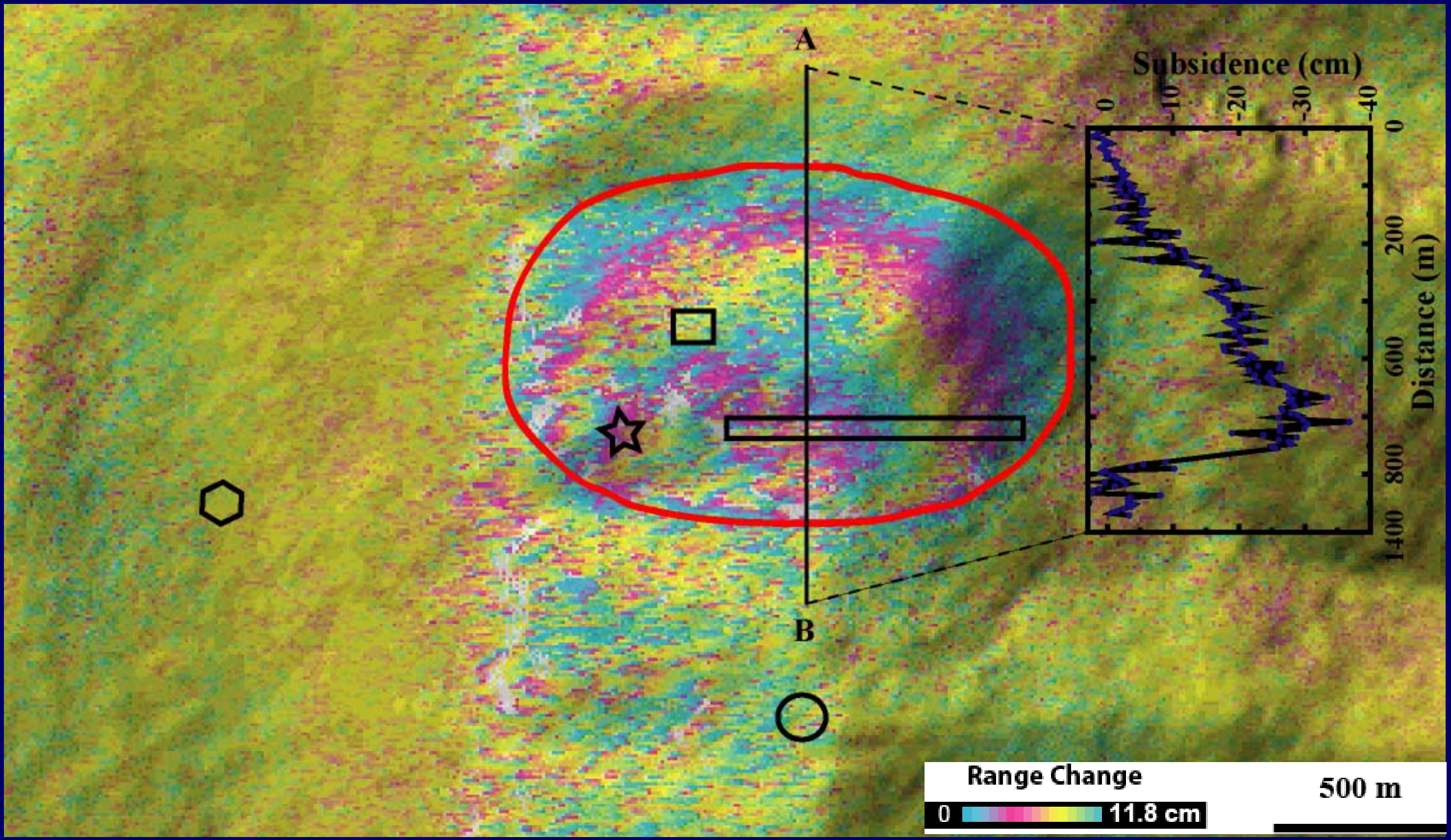
Ground Surface Deformation From InsAR
Co-event: 06/08/2007 – 09/08/2007



USGS

Co-event: 06/08/2007 – 09/08/2007

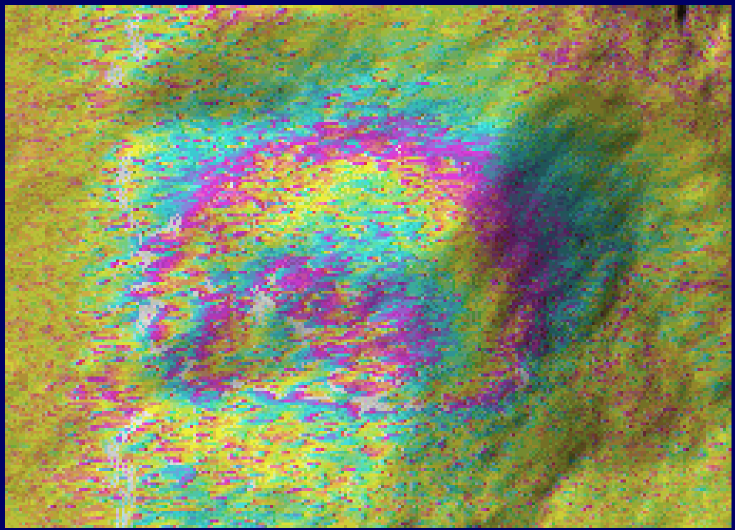


- ⬡ - the epicenter from the standard relocation program.
- ○ - the epicenter from a localized velocity structure.
- □ - the epicenter from the master-event method.
- ☆ - the epicenter from the double-difference relocation method.
- ▭ - the damaged area by the MSHA

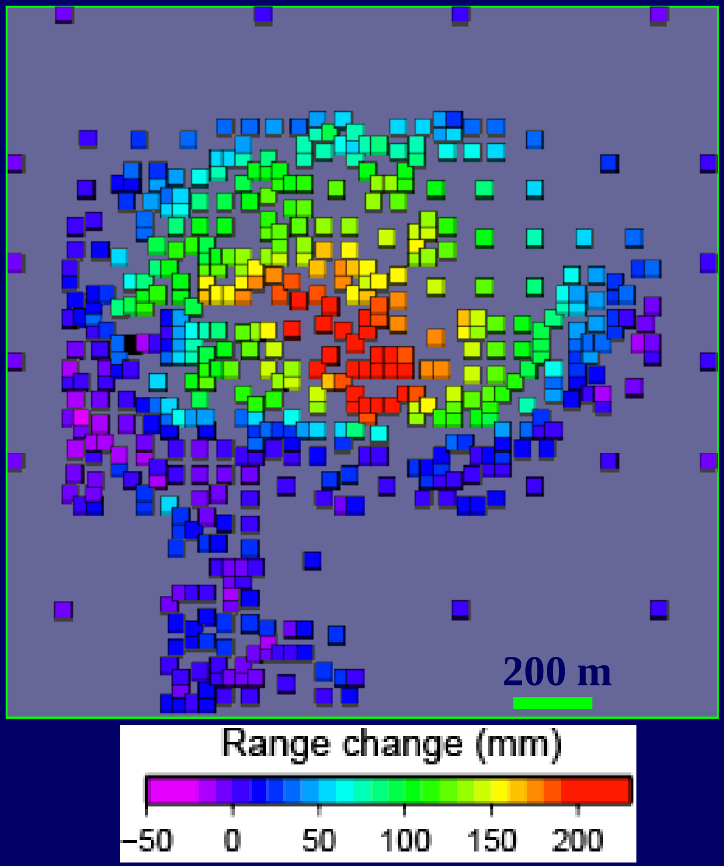
USGS

Deformation Modeling

- The sharp break in phase gradient on the south edge of the deformation signal is an important observation that is diagnostic of more than just a simple collapse model for the deformation source.
- InSAR data are parsed using a quad-tree algorithm.
- Deformation is modeled with distributed dislocation (Okada) sources.
- An adequate model is defined as one for which the variance of the residual (observed data minus calculated) is reduced to the same variance as the noise in the non-deforming area of the interferogram.

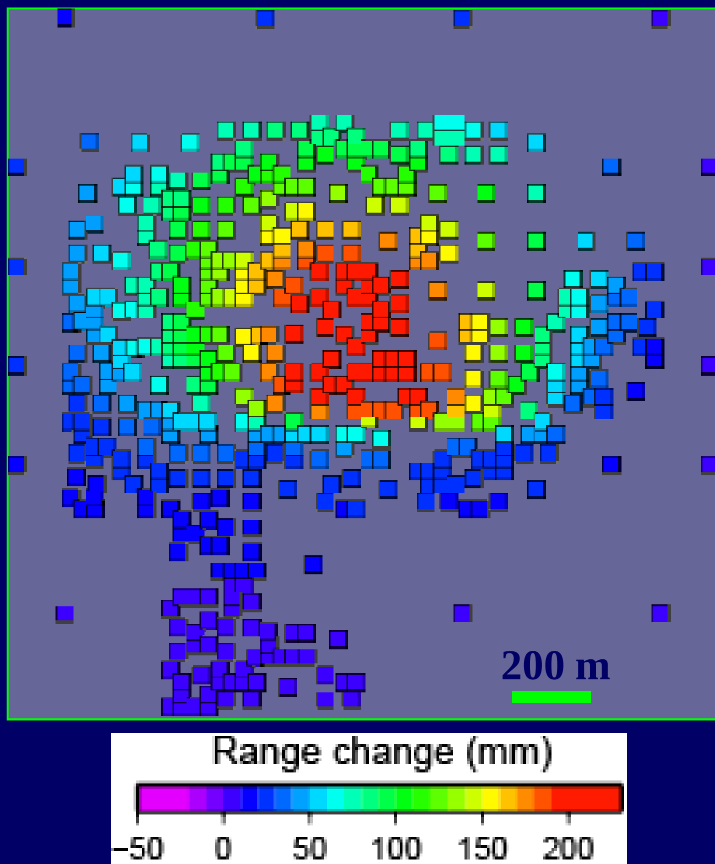


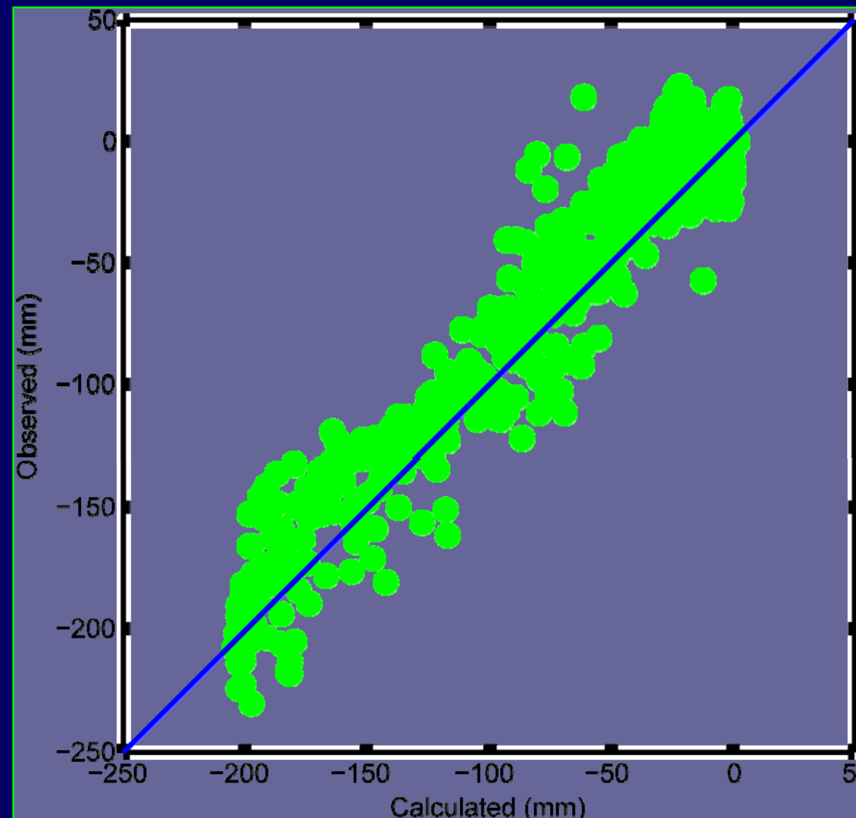
Observed Deformation



Modeled Deformation

Collapse-only sources





- **An adequate fit is only found where the depth of flat lying sources is less than ~100 m.**
- **The mine depth is know to be around 500 m.**
- **Therefore, a simple collapse model with spatially varying collapses cannot explain the deformation field seen in the interferogram.**

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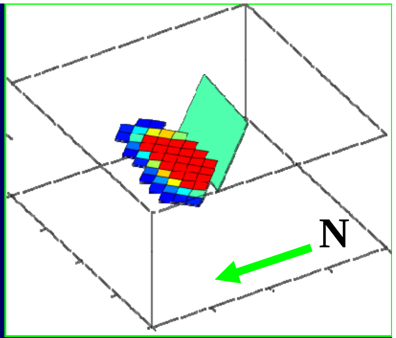
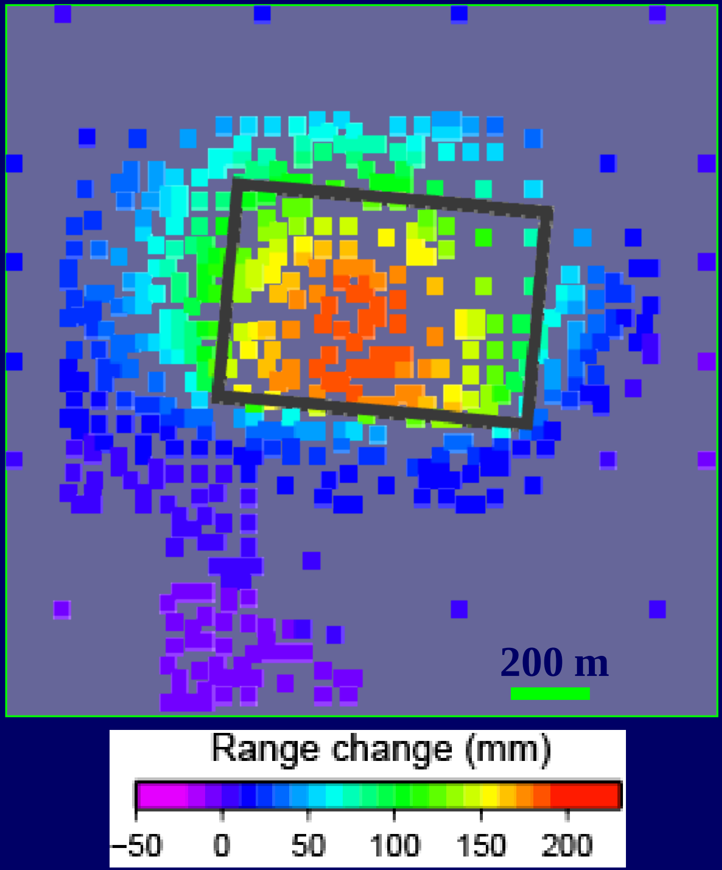
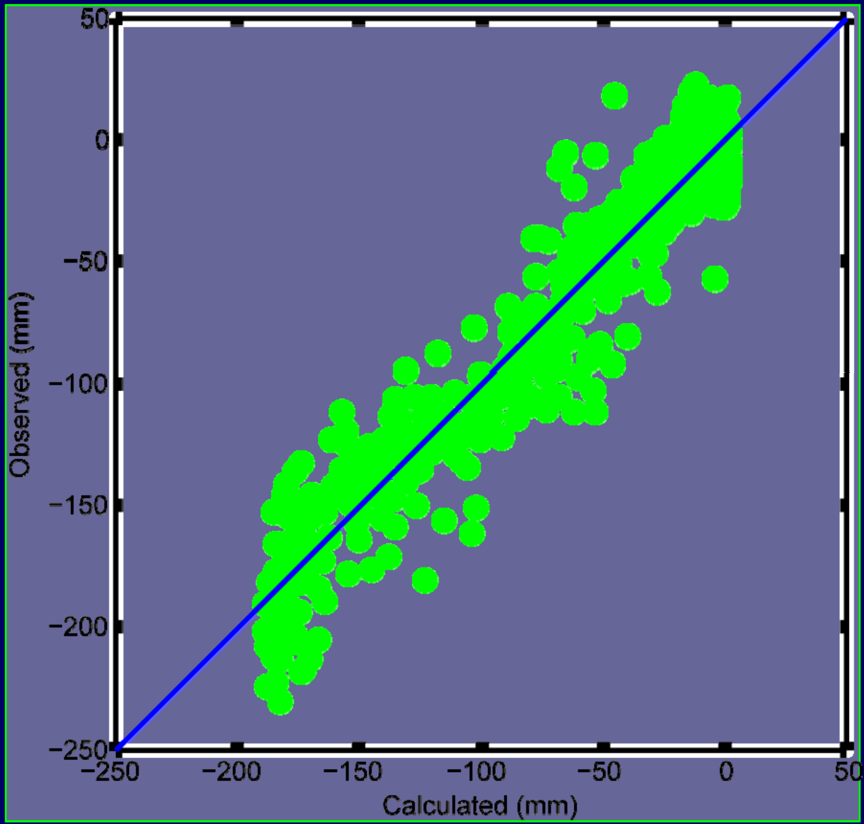
Modeled Deformation

Collapse sources + 40°-dipping fault







- constraining the depth of flat lying collapse sources to be 500 m
- adding a shallow uniform slip normal fault that dips to the north.

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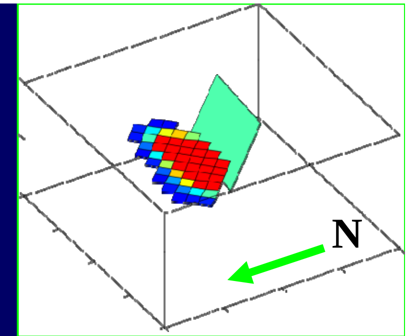
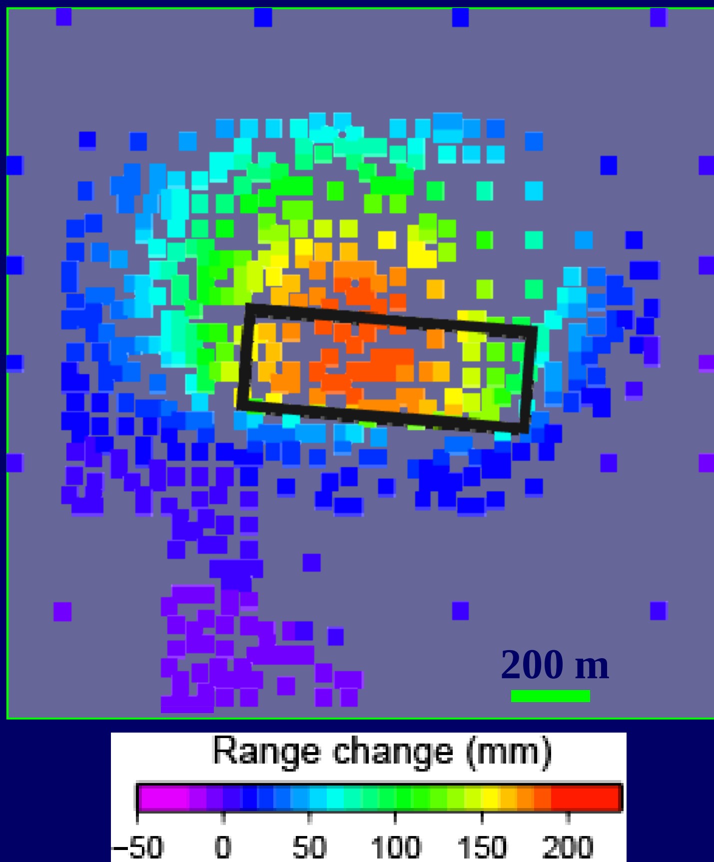
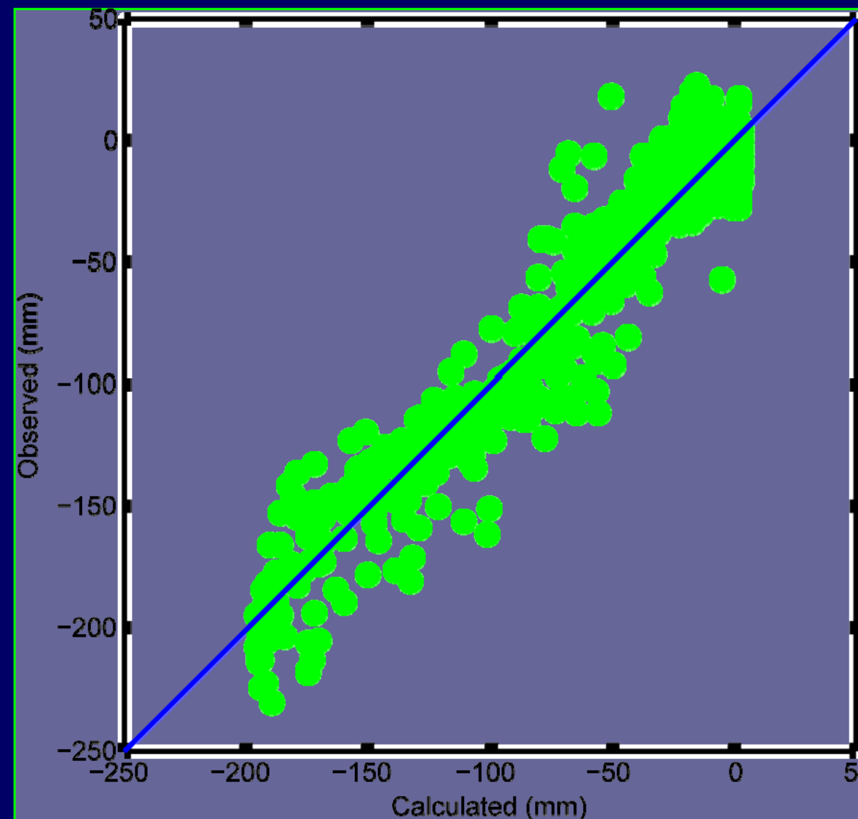
Modeled Deformation

Collapse sources + 65°-dipping fault







- constraining the depth of a flat lying collapse source to be 500 m
- adding a shallow uniform slip normal fault that dips to the north.

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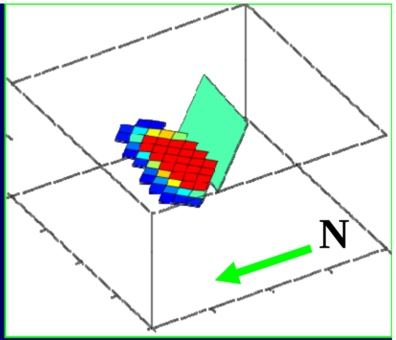
Modeled Deformation

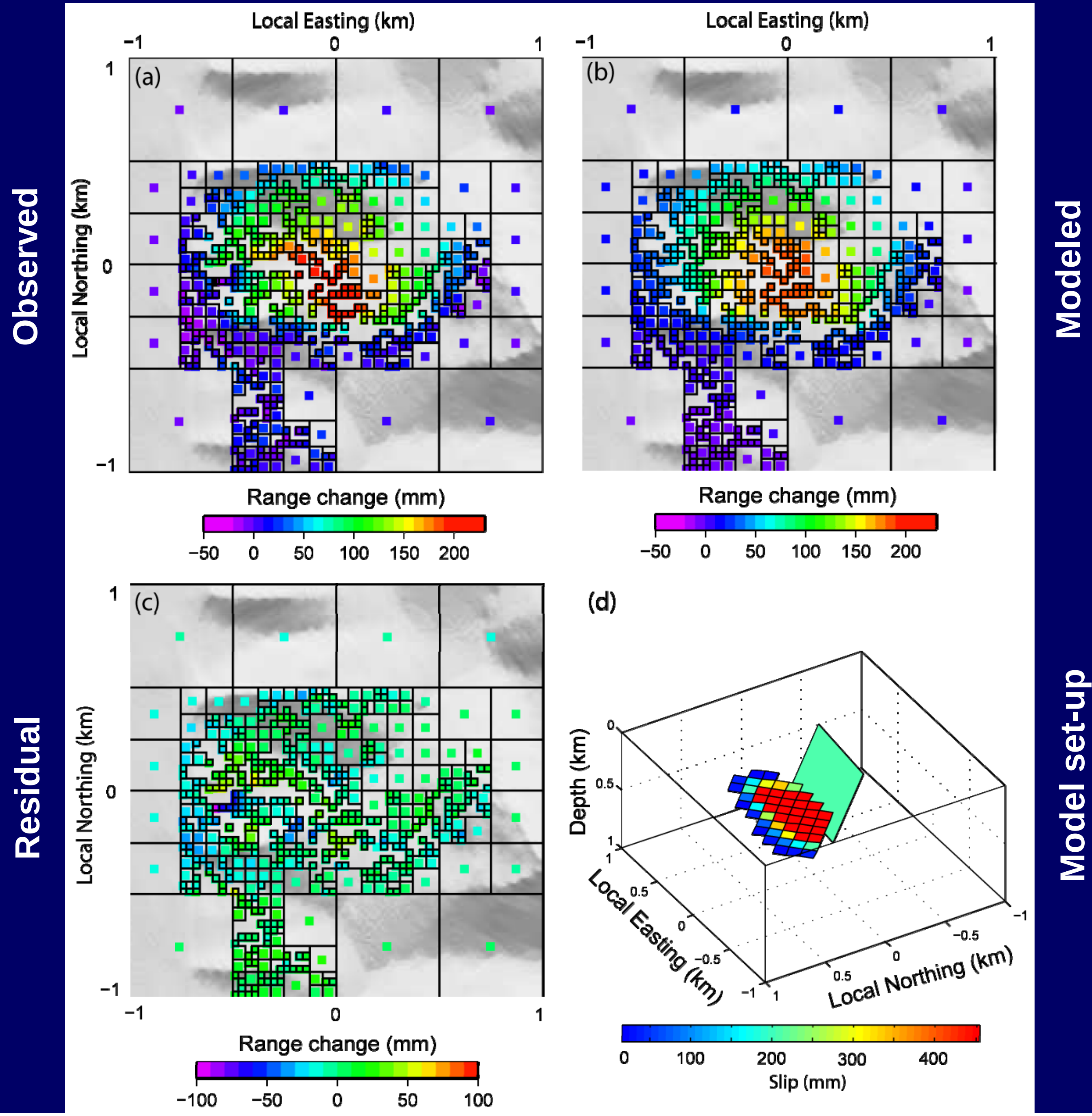
Collapse sources + a normal fault



- We cannot well constrain the dip of the normal fault component of the model.
- At the 95% confidence level, a dip between 10° and 85° provides adequate fit.
- The top of the fault is shallow, shallower than 70 m and deeper than 20 m.
- The ratio between the normal fault and the collapse component decreases from about 2.5 at 20° dip to 0.3 at a dip of 85°; however, a model with a dip of 85° for a normal fault is too steep to intersect the modeled collapse area.
- The estimated geodetic moment (Mw4.5) is larger than seismic moment (Mw4.1).

USGS

Our favorite model



Lu & Wicks, 2010

Springer

DZ's book

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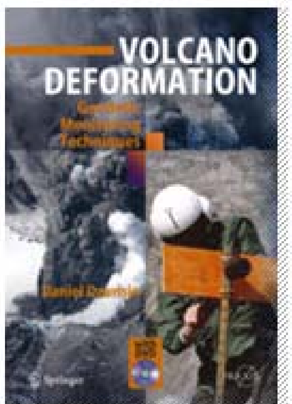

Volcano Deformation

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Dzurisin, Daniel

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