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Effect of Wind on the Rise Height of Volcanic Plumes

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Abstract. A theoretical model of a volcanic plume, based on applying the equations of motion in a plume-centered coordinate system, suggests that the interaction between a volcanic plume and wind causes enhanced entrainment of air and horizontal momentum, plume bending, and a decrease in plume rise height at constant eruption rate. Because of rapid dilution in the high windspeeds of the polar jet, plumes that vary over more than one order of magnitude in mass eruption rate (10^6 to $10^7 - 10^8$ kg/s), if injected into the polar jet, may all attain rise heights only slightly different from that of the core of the jet, ~ 10 km, as opposed to $17 - 33$ km in a still atmosphere.

1. Introduction

Volcanic plumes interact with the wind at all scales. On the smallest scales, the wind shapes local eddy structure. At larger scales, wind affects the entire plume trajectory. Small, tropospheric plumes are distorted or "bent-over" *Wright*, 1984 [@] even in moderate breezes by the addition of horizontal momentum, while large plumes that penetrate the stratosphere are bent-over only when windspeed is high *Sparks et al.*, 1997 [@].

The polar jets or jetstreams are regions of high windspeed that span the globe at latitudes from 30°to 60°. The jets mark the convergence zone between warm subtropical air and cold polar air. They are geostrophic winds and therefore are associated with a rapid change in vertical pressure gradient and tropopause height. The height of the tropopause is considerably lower on the poleward side of the jet than it is in the mid- and low-latitudes, averaging 9 km in winter (occasionally as low as 5 km in tropospheric folding events) and 10 km in summer, as opposed to 16 km at the equator *Glaze and Baloga*, 1996 [@]. Windspeeds within the jets average $\sim$ 40 m/s in winter, and $\sim$10 m/s in summer. Maximum speeds estimated in the core of the strongest jets are as high as 130 m/s. In cross section, windspeed decreases with height and horizontal distance from the core of the jet, which occurs at $\sim$ 10 km altitude. Jet width (in planform) varies up to hundreds of kilometers, whereas jet thickness is as little as one kilometer.

Even large volcanic plumes that are injected into the atmosphere at high latitudes can be expected to sometimes show profound effects caused by the interaction of the plume with the jet. In fact, jet speeds can approach and even surpass local plume speeds, resulting in ingestion of unusual amounts of air during plume rise, bending over of the plume in the windfield, and subsequent effects on maximum plume height and tephra dispersal. Because of this possibility of interaction of even large plumes with wind in eruptions at high latitude, the present contribution develops a plume model that incorporates some of the potential effects of wind in general, and of the plume-jet interaction in particular.

2. Model of Plume Motion in Wind

The relative importance of wind entrainment to plume bending is a complex issue that has been addressed by numerous workers interested in the movement of plumes generated by smokestacks and other engineering outflows into windy air *Wright*, 1984 [@]. The following analysis builds upon these engineering results and adapts their findings to the much more vigorous volcanic case.

2.1. Coordinate System

We can construct a quantitative integral model for plumes that entrain mass and momentum from the wind *Hewett et al.*, 1971 [@]; *Wright*, 1984 [@], by tracking the development of the plume in a plume-centered coordinate system. In the following analysis, x is the horizontal downwind direction, z is the vertical direction, and s represents a local coordinate tangential to the plume axis. Theta, ϑ, is the coordinate expressing the inclination of the plume centerline to the horizon. The equations expressing the coordinate transformation between (x, z) and (s, ϑ) are given by:

$$z = \int \sin\vartheta ds \tag{1a}$$

and,

$$x = \int \cos\vartheta ds \tag{1b}$$

2.2. Equations of Plume Motion

In plumes that are significantly affected by a crosswind, the entrainment velocity, U_ϵ, must be a function of windspeed, V, as well as axial plume speed, U. Numerous workers have investigated the use of different entrainment-velocity relationships *Wright*, 1984 [@]. Reasonable correspondence between one such entrainment relationship and experimental data has been obtained *Hewett et al.*, 1971 [@]:

$$U_\epsilon = \alpha|U - V\cos\vartheta| + \beta|V\sin\vartheta| \tag{3}$$

where $\alpha|U - V\cos\vartheta|$ is entrainment by radial inflow minus the amount swept tangentially along the plume margin by the wind, and $\beta|V\sin\vartheta|$ is entrainment from wind; α is the radial entrainment parameter, and β the wind-entrainment parameter. Equation 3 assumes that the magnitude of the horizontal wind component is much larger than the vertical component. Based on laboratory experiments, α and β have been shown to be constants equal to ~ 0.15 and unity, respectively *Hewett et al.*, 1971 [@]. Near the vent, where plume density may be five times that of the ambient atmosphere *Sparks et al.*, 1997 [@], and plume decompression occurs in the crater *Woods and Bower*, 1995 [@], the entrainment parameters may vary from these values. However, all models of vertical plumes suggest the parameters should approach these values within a kilometer or so of the source *Sparks et al.*, 1997 [@].

The equations of motion *Hewett et al.*, 1971 [@] have been rederived for the volcanic case, to account for realistic sedimentation of pyroclasts and for the large density differences between plume and ambient medium that occur in volcanic plumes *Ernst et al.*, 1996 [@]; *Glaze and Baloga*, 1996 [@]. For mass conservation (continuity), we have:

$$\frac{d}{ds}\left(\pi b^2 \rho U\right) = 2\pi\rho_a b U_\epsilon + \sum_{i=1}^{N} \frac{dM_i}{ds} \tag{4}$$

where b is characteristic plume radius, ρ is bulk plume density, ρ_a is ambient atmospheric density and M_i represents the mass flux of pyroclasts of size fraction i within the plume. The first term on the right-hand side represents the gain in mass flux by entrainment of air, whereas the second term represents the loss of mass flux by fallout of pyroclasts. The equation for conservation of axial momentum is:

$$\begin{aligned}\frac{d}{ds}\left(\pi b^2 \rho U^2\right) &= \pi b^2 \Delta\rho g \sin\vartheta \\ &+ V\cos\vartheta \frac{d}{ds}\left(\pi b^2 \rho U\right) \\ &+ U \sum_{i=1}^{N} \frac{dM_i}{ds}\end{aligned} \tag{5}$$

where the first term on the right-hand side represents the change in momentum caused by the component of gravitational acceleration, g, in the axial direction, and the second term represents entrainment of momentum from wind. One effect of entrainment of wind is to generate a net horizontal plume momentum that does not exist in a still atmosphere. The conservation of the radial component of momentum is given by:

$$\left(\pi b^2 \rho U^2\right)\frac{d\vartheta}{ds} = \pi b^2 \Delta\rho g \cos\vartheta - V\sin\vartheta \frac{d}{ds}\left(\pi b^2 \rho U\right) \tag{6}$$

where the left-hand side represents the change in ϑ caused by the entrainment of momentum at an angle to the plume axis by both gravity (first term on right-hand side) and wind (second term). Note that the small-angle approximation is made for $d\vartheta$. The conservation of specific enthalpy is given by:

$$\begin{aligned}\frac{d}{ds}\left(\pi b^2 \rho U C_v T\right) &= 2\pi b U_\epsilon \rho_a C_a T_a - \pi b^2 U g \sin\vartheta \\ &+ C_p T \sum_{i=1}^{N} \frac{dM_i}{ds}\end{aligned} \tag{7}$$

The first term on the right-hand side is the energy added by the entrainment of air, the second term is the change in thermal energy by conversion to/from gravitational potential energy, and the third term is the loss of heat by sedimentation of pyroclasts. C_v is the bulk heat capacity of the material in the plume, T is its temperature, C_a and T_a are the heat capacity and temperature respectively of the air, and C_p is the heat capacity of the pyroclasts. The conservation of mass flux of particles for multiple grain size fractions, M_i, is given by *Ernst et al.*, 1996 [@]:

$$\frac{dM_i}{ds} = -\frac{p}{bU}\left(w_s - \frac{fU_\epsilon}{db/ds}\right)M_i \tag{8}$$

where p is a probability that an individual particle will fall out of the plume, f is an empirical re-entrainment parameter that is a function of plume strength and particle size, and w_s is the settling speed of a particle in the given size class (in the current model, $i = 1$ to 19 for pyroclasts between 10 and -8 ϕ at 1-ϕ intervals). The probability of fallout, p, is a function of plume geometry, and should have an approximately constant value of ~ 0.23. Because of the strong inflow toward the plume caused by wind and atmospheric entrainment, pyroclasts $<\sim 10$ cm are re-entrained at lower heights in a plume after falling from greater heights. The vast majority of particles are thus re-entrained, and therefore there can generally be little deposition of pyroclasts from the eruption column itself. Fitting a curve through experimental results *Ernst et al.*, 1996 [@], a reasonable form of the re-entrainment function, f, for a volcanic plume is:

$$f = 0.43\left(1 + \left[\frac{0.78}{F_0^{1/2}\mu_0^{-1/4}/w_s}\right]^6\right)^{-1} \tag{9}$$

In equation (9), F_0 is the specific thermal flux at the vent, $F_0 = b_0^2 U_0 C_{v0} T_0$, and μ_0 is the specific momentum flux at the vent, given by $\mu_0 = b_0^2 U_0^2$. This empirical relationship fits data for plumes that are not affected by wind, and probably does change as a plume becomes bent over. However, there are no data for the bent-over case, and therefore we have used the re-entrainment function for a vertical eruption plume. This form may overestimate the number of particles that are re-entrained, as bending over can be expected to lessen re-entrainment.

Equations 8 and 9 allow calculation of the total number of pyroclasts passing beyond any downstream distance within the plume. These pyroclasts are not distributed evenly within the plume in any given cross section. To calculate the concentration of pyroclasts at any radial position within the plume, we rely on the observation, frequently corroborated in experiment, that the time-mean concentration of any passive tracer or particle fraction within a plume has a Gaussian profile *Morton et al.*, 1956 [@]. Thus the concentration as a function of radial distance, $C_i(r)$, within a volcanic plume follows:

$$C_i(r) = \frac{M_i}{\pi b^2 U}\exp\left[-\frac{r^2}{b^2}\right] \tag{10}$$

2.3. Atmosphere

For purposes of the model, an inventory of the atmospheric wind speed, temperature variation and density variation must be kept. For results shown here, atmospheric temperature and density for the standard ICAN atmosphere are used. Various models of wind speed are used to test the effects of profiles in which the wind is constant with height, or a jet exists. Jets are simplified to have a Gaussian profile in vertical cross-section.

Using equations 4 through 9, and initial conditions for wind speed, plume radius, the axial plume speed, and the particle grain-size distribution, a differential model for the plume motion can be constructed. C_v, T and ρ must be updated at each time step according to algebraic relations discussed elsewhere *Sparks et al.*, 1997 [@]. Time-mean radial concentration profiles are calculated with equation 10.

3. Discussion

Because a plume is not a solid object, streamlines in the ambient crossflow are not merely diverted about it. Some of the crossflow is ingested by the plume to increase the apparent entrainment coefficient. The increased entrainment results in an increase in the dilution rate of the buoyancy of the plume, and therefore a decrease in the total rise height or eruption column height with increasing wind speed (Fig. 1). This decrease in rise height is most pronounced in small plumes, for which the mass of air entrained by wind represents a significant proportion of the total entrained air mass even close to the vent. For plumes with a larger radius, the proportion of air added by wind is smaller, hence the effect on plume height is less pronounced. Nevertheless, a given observed plume height cannot be directly used to calculate mass eruption rate without taking into consideration windspeed. For large eruptions and low windspeeds, the error introduced by not considering wind is small. However, even for large eruption columns with rise heights of 30 km, for example, realistic but high mean windspeeds through the atmosphere of ~ 25 m/s could yield eruption rate estimates that are an order of magnitude in error. Fig.

At sufficiently high wind speeds or low mass eruption rates, a plume can not only ingest significant quantities of air by wind, but also the centerline of the plume can become distorted or bent-over in the wind field (Fig. 2). A plume bends over because of the addition of horizontal momentum by the wind (equation 6). A typical bent-over plume does not have a constant radius of curvature along its entire path, even in a wind that is uniform with height. The radius of curvature increases with height in a uniform field because the fractional increase in the horizontal component of momentum flux Fig.

decreases as more of the wind's momentum is taken into the plume.

No plumes are erupted into atmospheres having a uniform wind speed with height. The most extreme case is perhaps the polar jet, from which most of the horizontal component of momentum is added to the plume in a relatively narrow height range aloft. The radius of curvature of the plume decreases with height into the jet, then increases as the plume traverses the jet on the upper side. Because of the high wind speed within the jet, and the low plume speed at height, the sudden encounter of the jet by the plume can result in a rapid ingestion of air and dilution of buoyancy. Plume height can thus be lowered dramatically, and in fact, plumes rise to heights about equal to the height of the jet over a range of mass eruption rates (Fig. 1).

4. Conclusions

The main results of the interaction between plumes and wind are a decrease in plume rise height at constant eruption rate, and plume bending. Especially for conditions of high windspeed or low mass eruption rate, wind should be considered among the several important variables that affect column height and tephra dispersal *Sparks et al.*, 1997 [@]. For eruptions into the polar jet, plume height remains approximately constant over a wide range of mass eruption rates. Jet speed and geometry vary widely; nevertheless, for reasonable values of these parameters (Fig. 1), volcanic plumes in the range between $\sim 10^6$ kg/s and $\sim 10^7$ to $\sim 10^8$ kg/s all rise between 9 and 11 km. In a still atmosphere, the rise height over this range of mass eruption rate increases from ~ 17 km to ~ 33 km.

Notation

C_a	heat capacity of air, 1000 J kg^{-1} K^{-1}
C_i	mass concentration of particles in i^{th} grain-size fraction
C_p	heat capacity of pyroclasts, 1100 J kg^{-1} K^{-1}
C_v	bulk heat capacity of plume material
F	specific thermal flux, b^2UC_vT
μ	specific momentum flux, b^2U^2
M_i	mass flux of i^{th} particle-size fraction
T	temperature of plume
T_a	temperature of air
U	axial plume speed
U_ϵ	entrainment speed
b	characteristic plume radius
f	particle re-entrainment parameter
g	gravitational acceleration, 9.807ms^{-2}
h	fall-out height of a pyroclast
p	probability of fallout
r	radial coordinate
s	downstream coordinate
w_s	particle settling speed
x	horizontal coordinate
z	vertical coordinate
α	radial entrainment coefficient, 0.15
β	wind-entrainment coefficient, 1.0
ϵ	entrainment coefficient
ρ	bulk plume density
ρ_a	atmospheric density
ϑ	angle between plume axis and horizon

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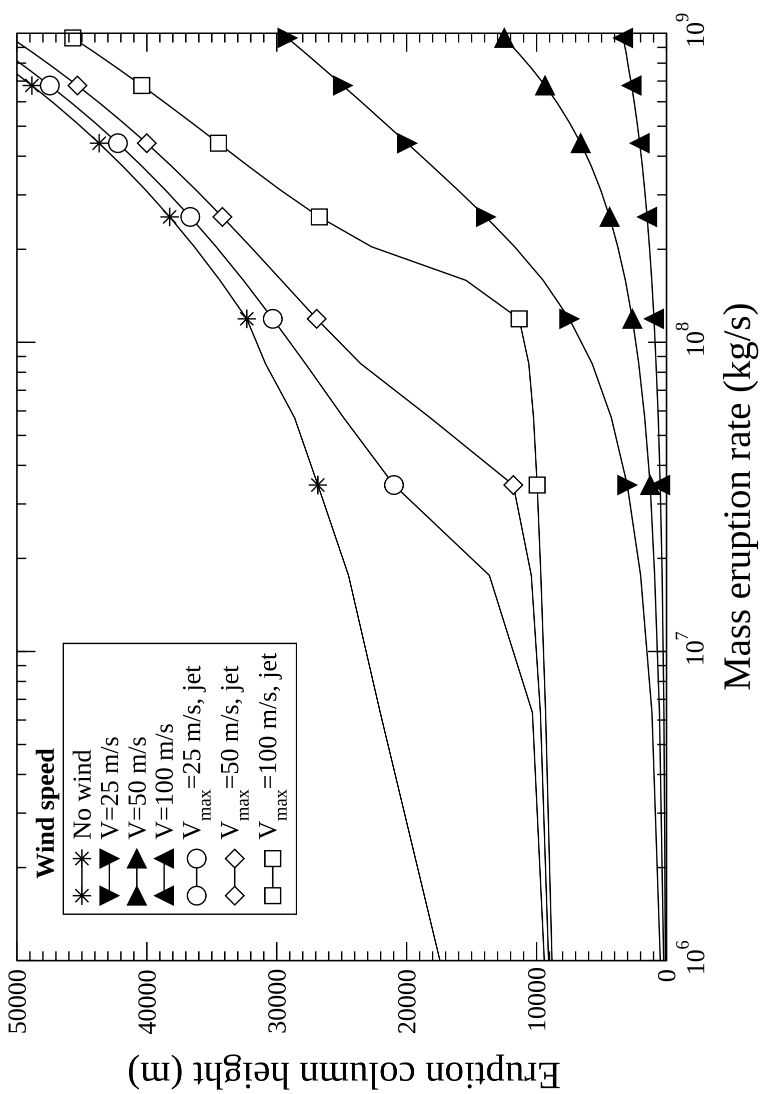

Figure 1. Eruption column height as a function of mass eruption rate under conditions of a jet aloft or constant windspeed with height. For clarity, not all points at which a calculation was done are shown. The model windspeed profiles for the jet cases were $V = V_{max} \times \exp(-(z - 10\,\mathrm{km})^2/(1\,\mathrm{km})^2)$. Thus, the jet is assumed to have a Gaussian profile, with a maximum speed at 10 km. Wind decreases plume rise height considerably. With a jet, over a wide range of mass eruption rate, plume height remains relatively constant because of the rapid ingestion of air aloft.

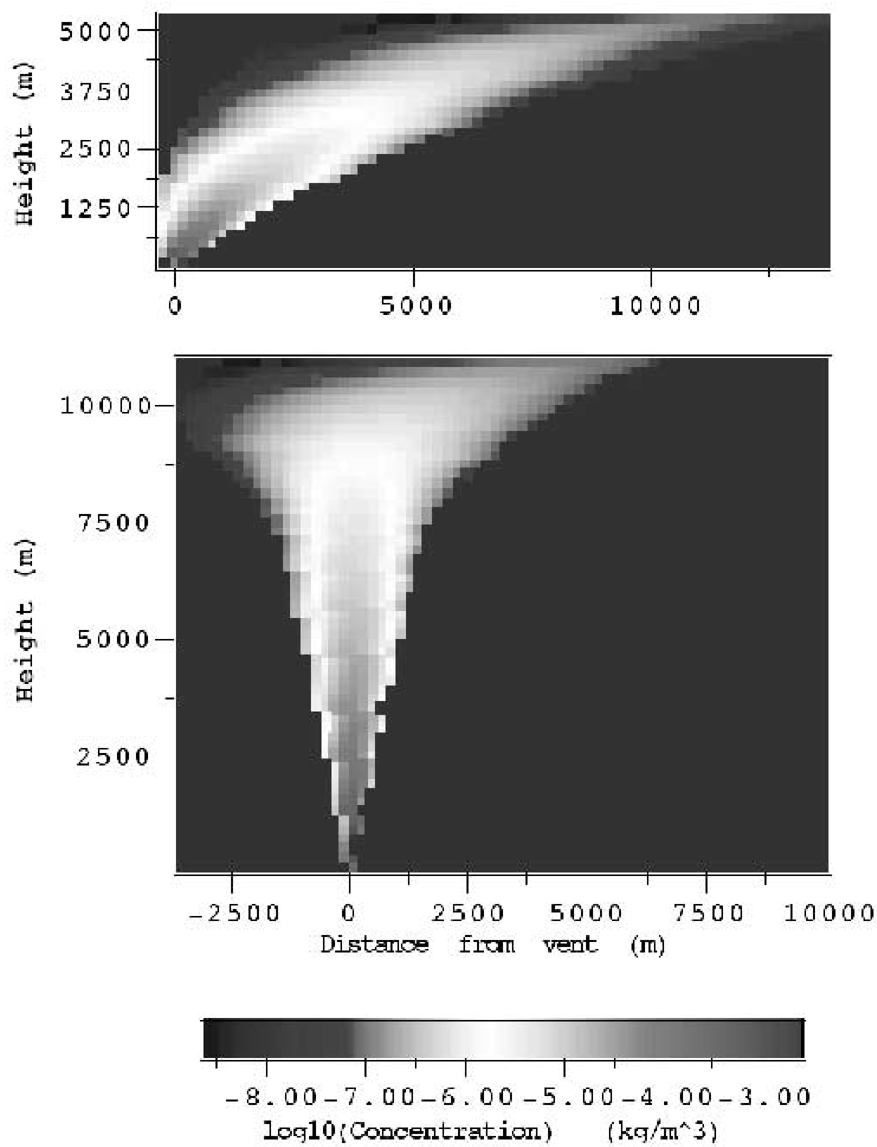

Figure 2. Time-mean concentration of the 8 mm grain-size fraction within a plume, illustrating plume bending. a) High mean windspeed (50 m/s), bent-over plume. b) Plume erupted into jet at height, showing effects of entrainment of horizontal momentum in a narrow height range. Eight mm grain-size fraction was chosen for visualization purposes only; any other size fraction would also effectively visualize the plume.