

# **Volcanic Seismicity**

Source processes and implications of VT, tremor, LP, VLP and tilt signals at volcanoes



# OUTLINE

1.Pre-eruption earthquake swarms

- 1.1. Characteristics and variability
- 1.2. Use for eruption forecasting
- 2. Event classification and source characterization
  - 2.1. What are the causes of different signal types
  - 2.2. Path distortions
- 3. Emerging methodologies
  - 3.1. Ambient noise
  - 3.2. Ground tilt from seismometers

# WHAT IS THE GOAL OF SEISMIC MONITORING OF VOLCANOES?

- Eruption forecasting?
  - Look for patterns in activity that have been seen elsewhere?
  - Model the physics in the conduit using seismic (and complementary) data

# **ERUPTION FORECASTING**

 Nearly every eruption is preceded by an increase in seismicity an earthquake swarm



- Large main earthquake
- Predictable duration of aftershocks based on mainshock magnitude
- Foreshocks are atypical

- No distinct mainshock
- Gradual increase and decay of activity
- Duration unpredictable (hours to years)

# **ERUPTION FORECASTING**

- Nearly every eruption is preceded by an increase in seismicity an earthquake swarm
- Relatively simple analyses can inform about the likelihood of an eruption
  - Event counts (as in previous slide)
    - Track number of earthquakes per day
    - Measure magnitude and examine energy release over time
  - RSAM
    - average absolute amplitude over time (1 min, 10 min)
    - includes all types of signals (tremor, VT, LP earthquakes)
    - no need to count individual events (may not be possible!)

#### REAL-TIME SEISMIC AMPLITUDE MEASUREMENT (RSAM)



Moran et al., USGS, 2008

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Moran et al., USGS, 2008

• Analysis of hundreds of pre-eruption earthquake sequences by McNutt and others suggested some common features



Shear fracture of host rock driven by

- ▶ Pressure increase in magmatic system
- Dike intrusion
- Duration may reflect
  - Depth of magma chamber
  - ►Type of fluid intrusion

- Temperature difference between host rock and magma
- Migration from shallow to deep sometimes seen





- Shallow intrusion
  - Gases exsolve from magma
  - Greater impedance between magma and host rock
- Magma may interact with ground waters
  - Steam may cause LF and tremor





#### Shallow intrusion

- Gases exsolve from magma
- Greater impedance between magma and host rock
- Magma may interact with ground waters
  - Steam may cause LF and tremor

- Relative quiescence
  - Gas venting to the surface through cracks
  - ▶ Temporary drop in pressure





- Explosive eruption
  - Rapid pressure changes
  - Magma flow
- Broadband seismic signals
  - ▶ HF to VLP
  - Infrasound





- Relaxation of the crust around the deep magma chamber causes earthquakes
  - may have different source mechanisms than pre-eruption events





# How to Improve Eruption Forecasting?

- We need to understand what various types of signals can tell us about the state of the volcano
  - Classify earthquakes
  - Model the source mechanism
  - Include other types of data
    - gas emission
    - infrasound
    - ground tilt





• Primarily defined on the basis of the frequency content



#### GENERAL VOLCANIC EARTHQUAKE CLASSIFICATION

- Explosion
  - broadband, long-duration signals resulting from pressure release, fracture, magma flow
- HF (high frequency) or VT (volcano tectonic)
  - tectonic earthquakes (slip on a fault)
  - Clear P and S wave arrivals
- LF (low frequency)
  - Typically no clear S wave
  - Low frequencies due to source and/or path
  - Includes long-period earthquakes, which occur at all depths within the crust
  - Tremor Semi-continuous signal with harmonic or irregular signals
- Hybrid
  - Characteristics of both HF and LF
  - Usually have high-frequency onset, low-frequency coda
  - Could represent a small VT that triggers an LP
  - Sometimes distinguished from LP on the basis of mixed first-motion polarities
    - suggests the event involves slip on a fault, start as a VT
- VLP (very-long-period)
  - involve volume changes, mass advection, drag forces

### CHARACTERISTICS OF HF (VT) EARTHQUAKES

- Brittle-failure events
- Due to slip on a fault, just like typical earthquakes in nonvolcanic areas
- Migration of activity used for tracking intrusions
- Typically small magnitude
  - large faults do not develop in the heterogeneous volcanic edifice
  - Iarge b values
- Mechanisms can vary with time due to changing stresses
  - Have been used to infer dike intrusion

#### DIKE INTRUSION AND EARTHQUAKE MECHANISMS



- Mechanisms can vary with time due to changing stresses
  - Have been used to infer dike intrusion

#### GENERAL CHARACTERISTICS OF LF SEISMICITY

- May have harmonic/narrow band signal
- Typically attributed to fluid interacting with solid volcanic conduit walls
  - gas, liquid or more likely multiphase
- Nonlinear processes that vary with time
  - physical properties of the system evolve
- Transient or long-lived
- Path and site distortions can cause events to look like LF events

# LP (LONG-PERIOD) EARTHQUAKES

- Known by many names
  - ▶ LP, B-type, tornillo, ...
- Broadband onset
  - Frequencies from .2 to 15 Hz
  - trigger
- Decaying, harmonic coda
  - Frequencies .5 2 Hz
  - resonance



• Typically shallow (< 3 km), but can be very deep (upper mantle)

# PATH-DISTORTED LPS?

- Path can filter some frequencies and enhance others
- Shallow low-velocity layers can trap waves
  - prolongs the duration of the signal and may mimic LP coda characteristics
- Topography focusses and defocusses waves
  - Waves can be trapped beneath steep topographic features (hills and volcanic edifices)
  - If underlain by strong reflective layer, the signal can ring for 10s of seconds



# SYNTHETIC MODELING OF STRUCTURE



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### SOURCE VS. PATH

Mount St. Helens

- Site affects are unlikely to be identical at all stations
  - Examine spectra of the coda for one event at all stations for similarity
  - Common spectral peaks at all azimuths are unlikely to be cause by site affects



### SOURCE VS. PATH

Mount St. Helens

- Site affects are unlikely to be identical at all stations
  - Examine spectra of the coda for one event at all stations for similarity
  - Common spectral peaks at all azimuths are unlikely to be cause by site affects
- Caveat: crack model predicts variations in frequency content with azimuth



## SOURCE VS. PATH

#### Difficult to identify for shallow events

- One way to determine if LF signal is due to path or source is to examine different events (a VT and LP) that occur at about the same location
- Share the same path for most, so any differences attributed to source

seismic

station

- Mammoth Mountain example

D

- Stacked spectra from 7 stations
- Two events closely-spaced
- Differences unlikely to be path-only



Pitt & Hill, GRL, 1994



In this example from Mount St. Helens, all the stations record the same dilatational first motion
This cannot be a double-couple earthquake in which slip occurs along a planar fault

but, can result from a volume decrease.

Waite et al., JGR, 2008



 Large aspect ratio crack filled with magmatic or aqueous multiphase fluid

- fluid velocity (a)
- fluid density ( $\rho_f$ )
- rock velocity (α)
- rock density (ρ<sub>s</sub>)
- Z = a  $\rho_f / \alpha \rho_s$



# CRACK MODEL

- Large aspect ratio crack filled with magmatic or aqueous (multiphase) fluid
  - crack width and length on the order of 100s of m for crack width of 100s of cm
- Resonator due to large impedance contrast (Z) between solid crack walls and fluid
  - Z = fluid velocity X fluid density / rock velocity X rock density
  - Traps energy in the crack
  - Large impedance contrast -> long duration coda

## CRACK MODEL

- Candidate fluids are:
  - bubbly magma
  - steam
  - steam with fine particles (dusty gas)
  - crystal-rich magma
- Predictions about the rate of decay of the harmonic coda can be made for specific fluid types



Q<sub>r</sub> describes the signal attenuation due to radiation from the crack

- Low Q<sub>r</sub> means the coda decays rapidly
- High Q<sub>r</sub> predicts long-duration codas

- High  $Q_r$  (long coda) is best explained by a dusty gas
  - ► Dust ~1 µm
  - Only tested fluid that can produce long-lived coda with Q significantly greater than 100
- Low  $Q_r$  (short coda) results can be explained by a variety of fluid mixtures
  - Frothy basalt
  - H<sub>2</sub>O gas- CO<sub>2</sub> gas
  - Bubbly water
- Dominant frequencies are different!
  - Crack dimensions are the same
  - Only the fluid content has changed

## CRACK MODEL

•  $Q_r$  varies from 10 at Redoubt to 1000 at Kusatsu-Shirane and Galeras


## CRACK MODEL: IMPLICATIONS

- Because of the slow wave speed (slower than acoustic velocity of the fluid), LP resonant frequencies are possible for relatively small cracks
- Repetitive LP events imply a non-destructive source process
  - crack can be excited into resonance hundreds or thousands of times without being significantly altered
- Increasing LP activity may imply
  - higher pressure in the magmatic or hydrothermal system
  - increase flow rates

#### **CONDUIT MARGIN FRACTURE RESONANCE**

- At silicic volcanoes, a model involving resonance of fluid-filled cracks along the conduit margin may explain LP earthquakes
- Large strains at margin cause brittle failure in hot rock
- Pressure changes can trigger resonance in system of interconnected cracks
- LP events may increase in frequency and merge into tremor
  - suggests a common source mechanism for LP and tremor activity



Neuberg et al., JVGR, 2000

#### CONDUIT MARGIN FRACTURE RESONANCE

• Increased activity may indicate an increase in effusion rate



## VILLARRICA LPS AND TREMOR

- weak seismic signal associated with bubble burst at the surface of the lava lake
- repeatable for years
- stack has high S/N



# VILLARR a)

- As with Soufriere Hills, tremor seems to have the same mechanism as the LPs
- The LP coda grows with increased distance from the source



# VOLCANIC TREMOR

- Long-duration signal with emergent onset
- No clear P or S arrivals
- May be dominantly surface waves or body waves
- Two types
  - Harmonic
    - spectral characteristics similar to the coda of an LP
    - may have multiple overtones indicative of a resonant source process
  - Non-harmonic
    - typically low-frequency and narrow band, but without harmonics

## HARMONIC TREMOR

- Narrow-band, longduration signal
- 1 or more (>10)
   harmonic overtones
   of the fundamental
   frequency, f<sub>0</sub>



# HARMONIC TREMOR

- A simple harmonic resonator (organ pipe) model:
  - $f_0 = v/2L$  (Hagerty, 2000)
    - where v is the speed of the interface wave
    - *L* is the length of the resonator
- nonlinear change in density at gas exsolution front acts as lower boundary

0 exsolution depth gas fraction gas fraction with depth to resonator <sup>></sup>ressure-dependent gas boundary L Decreased lower as acts

#### HARMONIC TREMOR GLIDE, FUEGO GUATEMALA



- Harmonic tremor with up to 10 harmonics
- Tremor typically glided upward just prior to an explosion over 1-2 minutes
- Fundamental frequency from 2 4 Hz
- Amplitude decayed as frequency increased

# MODEL FOR TREMOR GLIDE

- harmonics that are integer multiples of a fundamental frequency, f<sub>0</sub>, suggest a column with matched boundary conditions (closed-closed or open-open)
- $f_0 = v/2L$



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  - rapid dissolution of existing bubbles?



# MODEL FOR TREMOR GLIDE

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- $f_0 = v/2L$
- for fixed *L*, increased *f*<sub>0</sub> implies increased *v* 
  - rapid dissolution of existing bubbles?
- for fixed v, increased  $f_0$  implies decreased L
  - pressurization of the magma due to sealing could cause exsolution front to migrate upward



# A MODEL FOR HARMONIC TREMOR GLIDE



- Gliding could be due to shortening effective length of the conduit
  - For v=400 m/s and f<sub>0</sub>=2, L=100 m => f<sub>0</sub>=4, L=50 m
  - Implies mobile boundary migrates about 0.4 m/s
- If conduit is sealing, increased pressure could reduce L and increase v

# ΑΜοι



## 10R GLIDE

#### TREMOR DUE TO FLOW INSTABILITY



- Rapid fluid flow through a constriction in the conduit can excite harmonic tremor
  - Consider the sound made by the slow release of air from a balloon
  - May work for liquid or gas
- Modeled for an incompressible, Newtonian fluid and elastic crack

#### **TREMOR DUE TO FLOW INSTABILITY**



- Modeled for an incompressible, Newtonian fluid and elastic crack (Julian, *JGR*, 1994)
  - Pressure difference (p<sub>1</sub>>p<sub>2</sub>) drives fluid through the constriction
  - walls close due to reduced pressure (Bernoulli effect)
  - narrower constriction reduces flow rate
  - walls open back up due to decreased flow velocity

## Some Bubble-Related Tremor Models

- Single bubble oscillation
  - frequency of oscillation depends on radius,
     r, and fluid pressure, P, and density, ρ:
- Bubble cloud oscillation
  - depends on gas fraction, β, dimension of bubble cloud, L:
- Increased number of bubbles, *N*, lowers the frequency:
- Example,
  - for r = 1 mm,  $f_o^{\text{single}} \sim 10,000 \text{ Hz}$
  - If  $N = 10^{12}$ ,  $f_o^{cloud} \sim 2 \text{ Hz}$

$$f_o^{\text{single}} = \frac{1}{2\pi} \sqrt{\frac{3P}{\rho r^2}}$$

(van Wijngaarden, 1972)





(van Wijngaarden, 1972; Chouet, 1996)

## NON-HARMONIC TREMOR MODELS

- Nearly all of the mechanisms described for harmonic tremor can also produce non-harmonic tremor under different conditions
  - system of cracks with different dimensions
  - heterogeneous magmatic fluid
  - fluid flow
  - oscillations of bubbles with many different sizes
- May be natural for some systems to switch between harmonic and non-harmonic tremor as condition change



## VERY-LONG-PERIOD EARTHQUAKES

- Observed at many active volcanoes having sufficient instrumentation
  - basaltic to dacitic
  - explosive and effusive
  - single pulse or oscillatory
- Likely involve fluid flow on much longer time scale than tremor or LPs
  - Mass advection and acceleration at places where conduit changes geometry
- Provide insight into conduit geometry and eruption dynamics

## VERY-LONG-PERIOD EARTHQUAKES

- Easy to identify
  - Use a low-pass or band-pass filter to remove the frequencies above the VLP range
  - If VLP signal is real, it will be largely filter independent



# VERY-LONG-PERIOD EARTHQUAKES

- Easy to identify
  - Use a low-pass or band-pass
     filter to remove the frequencies<sup>0.00</sup>
     above the VLP range
  - If VLP signal is real, it will be largely filter independent
- Pitfalls
  - step-response will look like VLP when filtered
  - filtered tilt can look like VLP signal



#### VLPS AT STROMBOLI



- Broadband signals associated with strombolian explosions at Stromboli
- Two types associated with two different vents
- Clearly have a VLP component in the unfiltered data

Chouet et al., JGR, 2003

#### VLPS AT STROMBOLI



TIME (MIN)

Chouet et al., JGR, 2003

#### VLPS AT MOUNT ST. HELENS



Waite et al., JGR, 2008

#### DETERMINING THE SOURCE PROCESS

- Invert the VLP seismic data for a representative set of forces
  - ▶ Seismic records convolutions of source mechanism, *m*, and Green functions, *G*

• 
$$u_n(t) = \sum_{i=1}^{N_m} m_i(t)^* G_{ni}(t), n = 1, ..., \text{ number of seismic traces}$$

•  $N_m$  is the number of mechanism components: 6 independent moment components + 3 single forces

 least-squares inversion based on this equation yields a best-fit location and mechanism for each event

- Interpret the forces in terms of realistic physical models
  - deformation of cracks, pipes, spheres
  - each of these has a mathematical representation
- Also consider forces associated with mass acceleration
  - the recoil force associated with vertical mass ejection
  - descent of magma around a large bubble
- Provides constraint on the geometry and dynamics

### VERY-LONG-PERIOD EARTHQUAKE ANALOG

- Bubble accelerates through the flair in the tube
- Liquid annulus falling around the bubble also must accelerate
- Net result is a force (F=ma) that can be translated to the surrounding rock



James et al, JGR, 2006





# SIGNAL FROM NOISE

- Studies in many settings are demonstrating that information can be extracted from ambient seismic noise
- Seismic waves scatter off of heterogeneities and result in many waves traveling in many directions
- By examining the signals from long recordings at station pairs or single stations, it is possible to model the scattering features in the medium
- Subtle changes in this over time may indicate changes in seismic velocities due to
  - pressure, temperature, etc.

# VILLARRICA AMBIENT NOISE

- Auto-correlated hour-long traces after removing large signals from earthquakes, etc.
  - correlation function varies over time



#### VILLARRICA AMBIENT NOISE



McKee and Waite, in prep

#### TILT FROM SEISMOMETERS

- the horizontal channels of seismometers respond to ground tilt because it accelerates the masses
  - vertical channels are less affected
- for very low frequencies, ground tilt signal dominates
- must be observed in the near field (< 2 km)</li>
- examples from Soufriere Hills [Voight et al., 1999], Merapi [Voight et al., 2000], Anatahan [Wiens et al., 2005], Stromboli [Genco and Ripepe, 2010], Meakan-dake [Aoyama and Oshima, 2008], Santiaguito [Johnson et al., 2009; Sanderson et al., 2010], Fuego [Lyons et al., 2012]

### ULPS AT FUEGO

- small explosions occurred approximately once per hour
- each explosion was preceded by a tilt signal observed on all stations within 2 km of the vent



#### SUMMARY

- Relatively simple measures of seismicity have been used successfully to predict many eruptions
- Research on different types of volcano seismic signals has helped to improve understanding of the processes involved at different preeruptive stages
- Integration with other data is a powerful new tool for understanding the physical processes
- New techniques have the potential to increase the warning time before eruptions



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